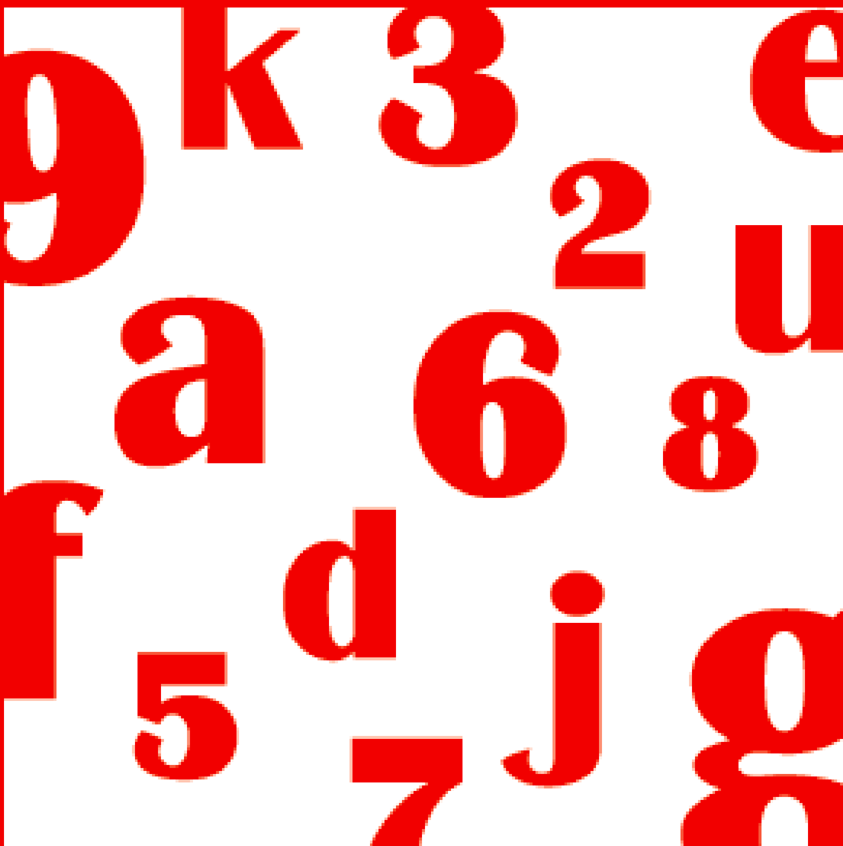
DATA INTERPRETATION

Calculation Based



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Index

Note:

Data Interpretation is not as much about theory as about skill in handling numbers. And no one approach fits all situations about interpreting data and doing the calculation bit. Hence consider the explanations given at the end of the book as an integral part of the book and your learning process. Most students look at the explanations ONLY for questions that they have got wrong or are not able to solve. We strongly advise you to go through the explanation of each and every question, even for the ones you have got right. Read through the various techniques given in the explanations to make calculation less tedious. And while CAT does not have a lot of calculation intensive questions, do not ignore this skill. Other exams like NMAT, IIFT, XAT and SNAP have calculation-based-Qs. All the best.

Growth Rates

One of the most common themes of questions in Data Interpretation is that of finding the growth rate. Growth rate is just a percentage increase (or a percentage decrease).

E.g. Sales of a company increased from Rs. 150,000 to Rs. 200,000. Find the growth rate of sales.

Think of the two values on a time frame – initially the sales was 150,000 and then it increased to 200,000 ……

$150{,}000 \longrightarrow 200{,}000$

In this case there is an increase in sales. The percentage by which 200,000 is more **than** 150,000 is the growth rate in this case. As learnt in percentages, the required growth rate will be $\frac{200000-150000}{150000} \times 100 = \frac{50{,}000}{150{,}000} \times 100$ i.e. 33.33%

E.g.: It was also quite possible that the sales of a company could have decreased from 200,000 to 150,000.

Would the growth rate in this case be the negative of the earlier answer i.e. –33.33%?

No, it would not be so. Hence it is important to think of the two values on a time frame ……

$200{,}000 \longrightarrow 150{,}000$

In this case there is a decrease in sales. Even this is called a growth rate, the difference being that in this case the growth rate will be negative. But in this case it will be the percentage by which 150,000 is less **than** 200,000 i.e. $= \frac{150{,}000-200{,}000}{200{,}000} \times 100 = \frac{-50{,}000}{200{,}000} \times 100$ i.e. –25%.

Finding Growth Rate

As seen in the above two examples, to calculate the percentage, the base will be the initial sales value. Thus, to generalise, if the sales of a company changed from A to B, then the growth rate would be $\frac{B-A}{A} \times 100$ %.

If $B > A$ (increase in sales), then it will be a positive growth rate.

If $B < A$ (decrease in sales), then it will be a negative growth rate.

With Multiple Years

In the above examples, there were only two sales values given. However in Data Interpretation usually a plethora of values would be present. One of the simplest data is regarding sales in a number of consecutive years, as given in the following table ……

Sales of Maruti Cars in India over 2002 – 06					
Year	2002	2003	2004	2005	2006
Sales ('000)	150	170	210	250	270

With such a data, different growth rates could be defined .....

A. Growth rate for a particular year:

What was the growth rate of sales of cars in the year 2006?

In such cases, when only one year is mentioned, it is assumed that the required growth rate is compared to the previous year.

Thus, in the year 2006, the sales grew from 250 (sales in the previous year) to 270. And the required growth rate will be $\frac{270-250}{250}\times 100 = \frac{20}{250}\times 100 = 8\%$

If data is not for consecutive years ...

In certain situations a periodic activity that is carried out at a regular frequency, may not be carried out every year e.g. census of population. Consider the following data:

Census:	1981	1991	2001
Population (in mn):	750	900	1100

What is the growth rate of population in the census of 2001?

Since the activity is conducted (or the data is provided) only at 10 year intervals, to find the growth rate of the population in census of 2001, we would not be comparing it with the population in year 2000, and since it is not given, the answer will not be "cannot be determined". In this case the comparison will be with the last data i.e. of the year 1991.

Thus, in the census of 2001, the population has grown by a percentage of

$$\frac{1100-900}{900}\times 100 = \frac{200}{900}\times 100 = 22.22\%$$

But remember this is NOT an ANNUAL percentage growth rate. Read ahead for the meaning of annual percentage growth rate.

B. Growth rate over a particular time period:

However if the question itself specifies a particular time-period, then appropriate initial and final values have to be taken to calculate the growth rate ......

By what percentage did sales grow over the years 2002 – 2005?

Please read the years carefully, because in such questions, many students just assume the question would be about the first value and the last value. In this case the question asks for the year 2005 and not 2006.

In the year 2002, the sales were 150 and it grew to 250 in the year 2005.

Thus, the required growth rate will be $\frac{100}{150}\times 100 = 66.66\%$

C. Average Annual Growth Rate

Compare the following question with the one just solved: "By what average annual rate did sales grow over the years 2002 – 2005?"

This question is different because of the words: "average annual". It is very liable that one misses these two words altogether and finds the total growth rate as found earlier. So read the question very carefully.

To find the average annual growth rate, we find the total growth rate of the sales over the entire time period given. Next we 'average' out this total growth rate over the number of years elapsed in the given time period to find the 'annual' rate

The total growth rate over the period 2002 – 2005, as found above is $\frac{250-150}{150}\times 100=\frac{100}{150}\times 100$

= 66.66%

Error-prone area: Number of years elapsed in a time period

In the time period 2002-2005, the number of years elapsed is 3 (and NOT 4). This is one another area in which beginners make an error. Growth happens over an entire year. Thus, in the period 2002-2005, the growth would have occurred in the annual years 2002-2003 and then 2003-2004 and lastly 2004-2005. These are 3 elapsed years.

It is wrong to say that over the time period 2002-2005, the number of years elapsed is 4 viz. 2002, 2003, 2004 and 2005. The sales of the year 2002 is the base value. It increases over the year 2002-2003. This is just one annual time period. And not 2 years.

Since this growth of 66.66% is achieved over 3 annual period elapsed, the average annual growth rate of sales over the years 2002-2005 is $\frac{66.66\%}{3}=22.22\%$

The intermediate years do not matter

If you have followed the above calculation of finding the average annual growth rate over the years 2002-2005, you would have noticed that the sales in the intermediate years, 2003 and 2004, do not matter at all. In the intermediate period the sales could have increased or dipped at any rate, we are only concerned with the net result: sales in 2002 was 150 and finally in the year 2005 was 250. We are not concerned about how volatile the sales were in the intermediate years.

Consider the census population data given in the box above, reproduced here again:

Census:	1981	1991	2001
Population (in mn):	750	900	1100

What was the average annual growth rate of population over the years 1991-2001?

Since the question requires the "average annual" growth rate of the population, the answer will be the total growth rate over the years 1991-2001 divided by the number of years elapsed from 1991-2001.

The total growth rate is already found as $\frac{200}{900} \times 100 = 22.22\%$. The number of years elapsed from 1991 to 2001 is 10 (91-92; 92-93; ......; 2000-01 are 10 annual time periods).

Thus the required average annual growth rate is 2.22%

D. Compounded Annual Growth Rate (CAGR)

In the earlier cases of annual growth rate, we were averaging out the growth rate over the number of years elapsed. Very rarely, the question requires us to find the compounded rate of annual growth.

In this case we will have to use the formula of Compound Interest to find the rate: $A = P\left(1 + \frac{r}{100}\right)^n$.

In the above formula, P, the principal, will be the initial sales value (of the starting year) and A, the amount, will be the final sales value (of the last year). *r* would be the compounded rate to be found and *n* would be the number of years elapsed.

Re-writing the formula in terms of r: $r = \left(\left(\frac{A}{P}\right)^{\frac{1}{n}} - 1\right) \times 100$

With the above data of car sales, if we had to find the compounded annual rate by which sales grew over the years 2002-2005, the answer would be dependent on finding $\left(\frac{250}{150}\right)^{\frac{1}{3}} = 1.66^{\frac{1}{3}}$

There are no easy ways to find this and hence such questions are rarely asked. We can use certain approximation techniques that are explained in next topic.

Assuming we found $1.66^{\frac{1}{3}} = 1.185$, then the required growth rate would be 18.5%

CAGR and SAGR

The average annual growth rate is also called as Simple Annual Growth Rate, SAGR.

We already know that the amount at the end of *n* years at a rate of *r* % p.a. would be higher if it is case of compound interest as against case of simple interest. In simpler terms money grows at a faster pace in the case of compound interest.

However, say the same Principal is kept at compound interest and also at simple interest for n years. And after n years it is found that the Amount in both the cases was the same!

Obviously this is possible only when the two rates are different (if they were the same, amount in case of compound interest should have been higher). Inspite of money growing at a faster pace in the case of compound interest, yet the amount was the same in the two cases. This means that the rate of interest in the case of compound interest was lower than the rate of interest in the case of simple interest.

Thus, with the same two initial (P) and final (A) values, the CAGR is always less than the SAGR.

In the solved examples the CAGR was 18.5% and SAGR was 22.22% over the years 2002-2005.

In reality, a data interpretation set would have a plethora of data involved, even more than the data of sales values over the years 2002-06. In such cases, one just has to be careful about the two underlying values between which the growth rate is asked. So WHILE reading the question itself, one should highlight the two values between which the growth rate is asked. The following exercise requires you to find the growth rate in a relatively simpler data structure.

Consider the following data of break-up of exports of services ......

Export of Services across 4 sectors (all values in billion $)

	Travel	Transportation	Labour	Insurance
Jan	0.9	1.1	0.6	2.0
Feb	1.4	0.9	0.9	1.2
Mar	2.2	1.1	1.4	2.4
Apr	2.5	1.1	2.1	2.8
May	2.0	1.6	1.8	2.6

1. Over the period Jan to May, which of the four service sectors – travel, transportation, labour and insurance, has shown the least percentage growth in their exports?

   1. Travel   2. Transportation   3. Labour
   4. Insurance   5. Both Transportation & Insurance

2. What is the growth rate of total exports of the four sectors considered together in the time period Feb to Apr?

   1. 83.33%   2. 87.5%   3. 88.88%   4. 93.18%   5. 99.99%

3. Over the entire five months, Jan to May, by what percentage is the total export of labour services lesser than the total export of travel services?

   1. 16.66%   2. 18.88%   3. 20%   4. 22.22%   5. 24.44%

Solutions:

For Q 1: Growth rate of exports of travel = 1.1/0.9 = 122.22%

Growth rate of exports of transportation = 0.5/1.1 = 45.45%

Growth rate of exports of labour = 1.2/0.6 = 200%

Growth rate of exports of insurance = 0.6/2 = 3/10 = 30%

Thus, least percentage growth has been shown by the sector of insurance.

Before calculating check if there is an obvious answer

For each sector, just looking at the value in the first row and that in the last row should have made it evident that exports of travel services more than doubled and exports of labour services have tripled. Thus, they have grown by a very large percentage and would never be the answers. Thus, we would have to do just two calculations that for transportation and insurance.

Even this could have been avoided by noticing that growth in transportation is 0.5 and that in insurance is 0.6. But transportation grew from a very less base, 1.1, whereas insurance had a much larger base, 2.0. Thus, growth *rate* would obviously be less for Insurance.

For Q. 2 and 3: For such questions involving total across sectors or across time period, you just need to be think, if you have to add across the columns or across the rows.

In Q. 2 we would need to add all values in the row of Feb and that in row of Apr.

Thus, total exports of all 4 sectors in Feb = 1.4 + 0.9 + 0.9 + 1.2 = 4.4

And total exports of all 4 sectors in Apr = 2.5 + 1.1 + 2.1 + 2.8 = 8.5

The required growth rate = 4.1/4.4. Approximating it to 4/4.4 i.e. 10/11 = 90.90%, we know the correct answer has to be greater 90.90%. Thus correct choice is (4)

In Q. 3 we would need to add all values in the column of labour services and that in the column of travel services.

Total exports of labour services for entire period Jan – May = 0.6 + 0.9 + 1.4 + 2.1 + 1.8 = 6.8

Total exports of travel services for entire period Jan – May = 0.9 + 1.4 + 2.2 + 2.5 + 2 = 9

Thus, percentage by which labour services is less *than* travel services = 2.2/9 = 24.44%

Write down all added sums neatly at appropriate place

Whenever you add values in a row (or that in a column), it is a very fruitful practice to write down the sum at the end of the row (or at bottom of the column). It is very frustrating to do the addition again if another question requires it. You would save a lot of time, effort and frustration if you follow this practice religiously.

Exercise 1:

Annual Sales (in Rs. lakhs) of three cement companies A, B and C over the period 2003 – 2006 is given in the following table.

Company	2003	2004	2005	2006
A	260	300	345	375
B	1050	1270	1350	1465
C	680	725	815	880

Note: Assume these are the only three companies in the cement market.

1. What is the simple annual growth rate of sales of company C over the period 2003 – 2006?

   a. 7.5% b. 10% c. 22.5% d. 30%

2. By what percentage has the cement market grown over the period 2003 – 2006?

   a. 9% b. 12% c. 30% d. 36%

3. By what percentage was the sales of company B more than the combined sales of A and C in the year 2005?

   a. 14% b. 16% c. 18% d. 20%

4. Over the time period 2003 – 2006, by what percentage is the average sales of company C more than the average sales of company A?

   a. 42% b. 67.6% c. 114% d. 142%

5. If it is expected that the sales of company B in the year 2007 will increase by 15%, what is the expected sales (in Rs. lakh) of company B in the year 2007?

   a. 1685 b. 1665 c. 1705 d. Cannot be determined

6. If it is known that sales of company C increased by 37.5% in the year 2003, find the sales (in Rs. lakh) of company C in 2002.

   a. 485 b. 505 c. 495 d. Cannot be determined

7. By what percentage is sales of company A in the year 2006 less than the sales of company C in the year 2003?

   a. 45% b. 55% c. 67.5% d. 81%

8. Find the compounded annual growth rate of sales of company A over the period 2004 – 2006.

   a. 12% b. 10% c. 8% d. 15%

Calculations bothering you?

As you must have seen in the above exercise, we would have to very often find growth rates by performing calculations like $\frac{200}{680}\times100$, $\frac{182}{128}\times100$. If you found the calculations tough it is strongly advised that you read the text given in the appendix on calculation techniques before proceeding with the following exercise. The next exercise will have even tougher calculations.

Exercise 2:

Given below is the Profit and Loss account of an fictitious company, say M/s Fiction Ltd. Below the table are given the meanings of the various headings of the first column. The data refers to one financial year ending in the month given in the first row. All figures are in Rs. Cr.

	Mar '05	Mar '06	Mar '07	Mar '08	Mar '09
Total Income	205.38	302.2	601.26	878.95	1040.4
Total Expenses	184.15	256.4	513.89	765.44	933.23
Operating Profit	21.23	45.8	87.37	113.51	107.14
Interest	6.85	9.46	23.74	36.91	56.61
Depreciation	1.99	2.69	5.65	8.12	11.57
PBT	12.39	33.65	57.98	68.48	38.96
Tax	4	7.49	17.82	21.02	7.35
Net Profit	8.39	26.16	40.16	47.46	31.61

Operating Profit = Total Income – Total Expenses

PBT = Operating Profit – Interest – Depreciation Net Profit = PBT – Tax

1. What is the growth rate of Income in the year ending in Mar 2009?
   a. 15.2% b. 16.6% c. 17.4% d. 18.4%

2. What is the average annual growth rate of Net Profit over the period Mar 05 to Mar 09?
   a. 69.2% b. 62.5% c. 55.3% d. 27.6%

3. If the expenses in year ending in Mar 2005 grew by 15%, what were the expenses of M/s Fiction Ltd. in the year ending in Mar 2004?
   a. Rs. 156.5 b. Rs. 160.1 cr c. Rs. 165.7 cr d. Rs. 170.2 cr

4. If Depreciation is supposed to grow at the same rate in the year ending in Mar 2010 as the rate by which it grew in year ending in Mar 2009, what will be the depreciation for the year ending in Mar 2010?
   a. Rs. 13.5 cr b. Rs. 15 cr c. Rs. 16.5 cr d. Rs. 18 cr

5. If both Income and Expenses grow by 10% in the year ending in Mar 2010, what will be the Operating Profit in that year?
   a. Rs. 121.2 b. Rs. 112.5 cr c. Rs. 117.9 cr d. Rs. 107.14 cr

6. In which year did Interest grow by the greatest percentage?
   a. Mar 06 b. Mar 07 c. Mar 08 d. Mar 09

7. Net profit is what percentage of PBT for the year ending in Mar 2007?
   a. 67.7% b. 77.7% c. 69.3% d. 81.1%

8. If Net Profit Margin (NPM) is the Net Profit as a percentage of Income, in which year is NPM, the highest?
   a. Mar 05 b. Mar 06 c. Mar 07 d. Mar 08

Successive Percentage Changes

When only Growth Rates are given and no underlying values

In the above exercises we were given the values (sales, exports, profits, etc) and then had to find the growth rates. Another type of data that one encounters often (and which is more error-prone area) is one where only the growth rates is given for successive years and we are not aware of the underlying values behind the growth rates ......

Exercise 3:

Growth rate of Sales of ABC Limited and that of XYZ Ltd. over the years 2005-2009

	Growth rate in Sales (%)				
Year	2005	2006	2007	2008	2009
For ABC Ltd.	10%	15%	–10%	5%	–8%
For XYZ Ltd.	–5%	12%	–10%	10%	5%

We do not know the actual value of the sales of ABC Ltd. All that we know is the growth rate. Using these growth rates, we can 'compare' the sales across the years. By 'compare' we mean

i. find the total growth rate over any period within the given range

ii. find the ratio of sales for any two years

Further if actual sales value for any one year is known, we can find the sales value for all the years. Let's learn how to do these with examples:

1. What is the approximate ratio of sales of ABC Ltd. In the year 2004 to the sales in the year 2006?

   a. 4 : 5 b. 2 : 3 c. 1 : 2 d. Cannot be determined

2. By what percent did sales of XYZ Ltd. grow over the period 2006-2008?

   a. No change b. 10.88% inc c. 1% decrease d. Cannot be determined

3. What is the ratio of sales of ABC Ltd. to the sales of XYZ Ltd. in the year 2007?

   a. 1 : 1 b. 1 : 2 c. 2 : 1 d. Cannot be determined

4. If the ratio of sales of ABC Ltd to that of XYZ Ltd. in the year 2007 was 2 : 1, find the approximate ratio of their sales in the year 2009.

   a. 3 : 2 b. 5 : 3 c. 9 : 5 d. Cannot be determined

5. If the ratio of sales of ABC Ltd to that of XYZ Ltd. in the year 2007 was 2 : 1, find the approximate ratio of their sales in the year 2005.

   a. 4 : 1 b. 3 : 2 c. 2 : 1 d. Cannot be determined

6. If the sales of XYZ Ltd in year 2007 was Rs. 150 cr, find the difference in the sales values of year 2005 and year 2009.

   a. 24.5 b. 20.2 c. 16.6 d. Cannot be determined

Solutions:

Q. 1: Even though we do not know the sales or the growth rate in the year 2004, yet the question can be answered.

If the sales in year 2004 was x, then the sales in 2005 was 10% more than x i.e. $1.1x$. Further the sales in year 2006 were 15% more than the sales in 2005 and hence were $1.15 \times (1.1x)$ i.e. $1.265x$.

Thus, the required ratio is $x : 1.265x$ i.e. 1000 : 1265 i.e. 200 : 253, option (a)

The above explanation should have made it apparent that the sales increased successively by 10% and 15%. One could also have found the total percentage increase using the formula $a+b+\frac{ab}{100}$ (this formula is explained in percentage chapter in Arithmetic book). Thus, from 2004 to 2006, sales increased by a total of $10+15+\frac{10\times15}{100}$ i.e. 26.5%. Thus, if sales in 2004 was $100x$, then sales in 2006 would have been $126.5x$ and we get the same ratio as found earlier.

Q. 2: We know the growth rates of sales of XYZ Ltd. in 2006, 2007 and 2008 was 12%, –10% and 10%. So do we find the net effect of all these 3 growth rates? Not really.

The growth rate in the year 2006, 12%, is the growth rate in year 2006 over the year 2005. Since we need to find the growth rate over the period 2006 to 2008, we have to start with the sales values in 2006 and then proceed till the sales in year 2008. This will entail increasing the sales of 2006 by –10% and 10% successively. And this using the formula is $-10+10-\frac{10\times10}{100}$ i.e. 1% decrease. (Using multiplying factors, if sales in 2006 were $100x$, then in 2008 would have been $1.1 \times 0.9 \times 100x$ i.e. $99x$, a decrease of 1%)

Error-Prone Area

To find the total percentage growth from year 1 to year n, one should **NOT** include the growth rate of year 1 (since this is a growth rate over previous year)

Q. 3: Since sales of both companies decreased by 10% in the year 2007, coming to the conclusion that sales would be same in year 2007 is grossly wrong. Since we do not have any idea about the underlying values, we can make no comments about the comparison of sales across the two companies. Hence answer to this question will be 'cannot be determined'

Q. 4: Assuming the sales of ABC Ltd and XYZ Ltd as $2k$ and k, the sales in the year 2009 can be found as $2k \times 1.05 \times 0.92$ and $k \times 1.1 \times 1.05$. Thus, the required ratio will be $\frac{2\times105\times92}{1\times110\times105}$.

If reducing this ratio seems to be slightly bothersome, then one could have orally found the total growth rate over the period 2007-09 and used them. Sales of ABC Ltd would have increased over the given period by 5 – 8 – 0.4 i.e. –3.4%. And that of XYZ by 10 + 5 + 0.5 i.e.

15.5%. If the sales in 2007 were assumed as $200k$ and $100k$, then the sales in 2009 would have been $193.2k$ and $115.5k$ and the ratio would have been $\frac{193.2}{115.5}$, which can then be checked with the options and accordingly reduced. Rough approximation will be $\frac{193}{115} = 1\frac{78}{115}$ i.e. between 1.6 and 1.7, option (b)

Q.5: Compare this question with the previous question, Q. 4. Here from 2007 we have to work backwards to 2005. Alternately, we can assume the sales in 2005 as x and y respectively and then proceed to find sales in 2007 and use the given ratio.

Another point that this questions tries to drive in is that before starting work, please have a look at the numbers. We see than the sales in year 2007, for both the companies, grew by –10% . This means that the ratio of sales in the year 2006 would also have been 2 : 1. Thus, one does not really have to find the successive effect of two percentage changes.

Assuming sales in 2005 as $100x$ and $100y$, sales in 2006 would have been $115x$ and $112y$ and thus we have $\frac{115x}{112y} = \frac{2}{1} \Rightarrow \frac{x}{y} = \frac{224}{115}$. The ratio $\frac{224}{115}$ can then be reduced and approximated based on the given options. Rough approximation will be $\approx \frac{225}{115} = \frac{45}{23}$ i.e. a little less than 2, option (c).

Q. 6: If the sales of XYZ in 2005, were x, then $x \times 1.12 \times 0.9 = 150$ i.e. $x = \frac{150}{1.12 \times 0.9}$

And the sales in year 2009 will be 150 × 1.1 × 1.05.

One could do the calculation bit and then find the difference. This sort of text-bookish approach is what makes one think that a lot of calculations are involved in DI. And this sort of calculation also frustrates one. An obviously easier way for the calculation is to try to do some work orally as follows ...

Finding sales in 2005: From 2005 to 2007, sales increased by 12% and then decreased by 10%. A 12% increase and 10% decrease is effective increase of 12 – 10 – 1.2 i.e. 0.8% increase. Thus after an 0.8% increase, the sales became Rs. 150 cr. Thus sales before the increase, in 2005 would have been marginally less than 150 cr.

If we had to approximate sales in 2005, we could have though that since the two sales values is so close, we can also find 0.8% of 150 instead of the slightly smaller sales value in 2005. Since 0.8% of 150 is 1.2, the sales in 2005 would have been very very slightly more than Rs. 148.8 cr.

Finding sales for 2009: In 2008 the sales would have been 150 + 15 = 165.

This increased by 5% in 2009. Next, 10% of 165 is 16.5 and hence 5% will be 8.25. Thus, sales in 2009 would have been 173.25.

Thus the required difference has to be almost 23.25 + 1.2 i.e. 24.45. This will be enough to choose the correct option (If you are too worried about the error margin, have a look at the accurate answer, 24.4404)

Next, one exercise for you to practice.

Exercise 4:

The following table gives the growth rates in GDP (%) for 10 economies of the world over the period 2004 to 2008. Read the data carefully and answer the questions that follow.

Country	2004	2005	2006	2007	2008
Afghanistan	9.4	14.5	11.2	16.2	2.3
China	10.1	10.4	11.6	13.0	9.0
India	8.3	9.4	9.7	9.1	6.1
Japan	2.7	1.9	2.0	2.4	–2.5
Maldives	9.5	–9	19.1	6.6	5.2
Nepal	4.7	3.1	3.7	3.3	5.3
Pakistan	7.4	7.7	6.2	5.7	2.0
Russian Federation	7.1	6.4	7.7	8.1	5.6
United Kingdom	2.8	2.1	2.8	3.0	0.7
United States	3.6	2.9	2.8	2.0	0.4

1. Among the following countries, which country's GDP grew at the greatest rate over the period 2005 to 2008?

   a. India b. Maldives
   c. Russian Federation d. Afghanistan

2. What was the ratio of India's GDP in the year 2005 to that in the year 2003?

   a. 11 : 10 b. 13 : 11 c. 12 : 11 d. Cannot be determined

3. By what percentage did Pakistan's GDP increase over the period 2006-2008?

   a. 14.5% b. 11.4% c. 9.3% d. 7.8%

4. Among the countries for which the above data is given, which country had the highest GDP in the year 2007?

   a. Afghanistan b. China c. India d. Cannot be determined

5. How many of the following statements is/are false?

   I. Except two countries, among the given countries, the GDP of all the rest has shown a continuous increase over the period 2003-2008

   II. For none of the given countries is the GDP in the year 2008 lower than its respective GD in the year 2003?

   III. The GDP of USA is continuously declining over the period 2004-2008

   a. None b. One c. Two d. All three

6. In 2009, Japan's GDP should increase by what percentage so that it's GDP in 2009 is the same as it was in 2007?

   a. 2.44% b. 2.5% c. 2.56% d. 2.62%

7. For how many countries, among those given, was the GDP in year 2005 lower than the GDP in the year 2003?

   a. 6 b. 1 c. 0 d. Cannot be determined

8. If the ratio of GDPs of Afghanistan and Maldives in the year 2005 was 5 : 4, which of the following best describes the ratio of their GDP's in the year 2004?

   a. 1 : 1 b. 2 : 3 c. 3 : 4 d. 4 : 5

9. If all the countries are arranged from left to right in increasing order of the growth rate in their GDPs over the period 2004-08, which country will be fourth from the left end?

   a. United Kingdom b. United States

   c. Nepal d. Japan

10. If in the year 2005, Maldives's GDP is more than Nepal's, Maldives's GDP would have been atleast how much percent more than Nepal's in the year 2004.

   a.13.3% b. 12.8% c. 12.5% d. 11.6%

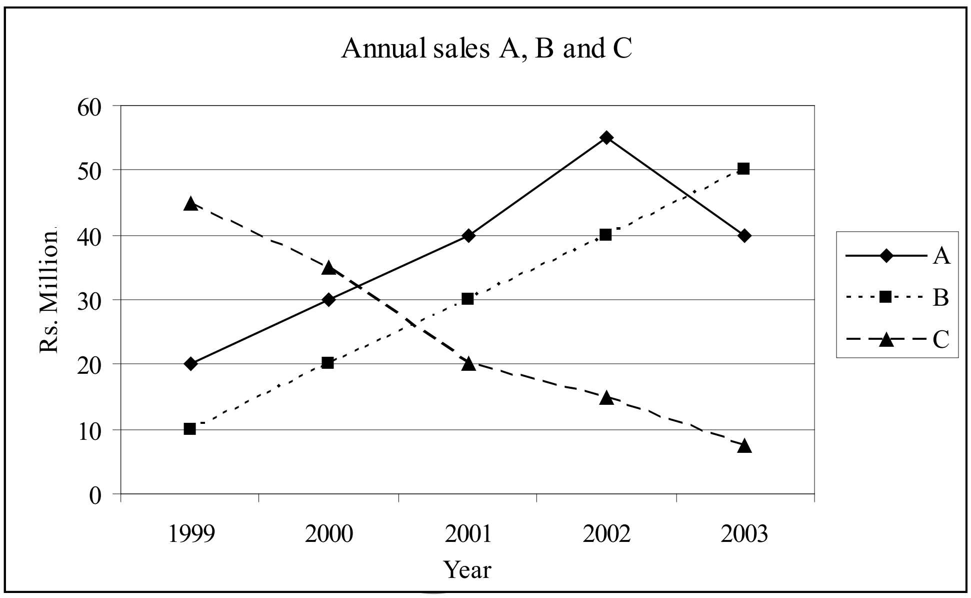
Growth rates and Line Graph

Is Slope exactly the growth rate?

When data is depicted as a line graph, comparing growth rates become all the more visual. However there is a very widespread error that is prevalent. So make sure you read the following carefully.

A line graph is a visual depiction of the data. Looking at the 'line', we can visually see how the underlying values have increased or decreased. A key characteristic of a line is its 'slope'. Slope is nothing but the 'inclination' of the line or the 'steepness' of the line. More is the inclination or steeper is the line, higher is the 'slope' of the line and vice versa.

It s commonly understood that the 'steepness' of the line is a function of the growth rate. Which to a certain extent is correct i.e. more steeper the line, the growth rate tends to be more and more flatter the line, growth rate tends to be less. However this is just indicative and need not be true in all circumstances. Look at the following graph and the subjective questions that follow it.



Q. 1: The sales of B is a straight line, i.e. the slope of each segment is same across all years. Does this imply that the growth rate of sales is same in all the years?

Q. 2: The slope of the line depicting sales of A and B is same for 1999 to 2001. Does this imply that growth rate of sales of company A and B is the same in the year 2000 and 2001?

Q. 3: The line representing sales of A in 2001-02 is steeper. Does this imply that company A witnessed the highest growth rate in the year 2002? If false, find the year in which A had the highest growth rate of sales.

Q. 4: The line segment of sales of C is steepest (in downward direction) in 2000-2001. Does this imply that company C had the highest growth rate (in negative sense) in 2001? If false find the year in the year in which sales of company C plummeted by the highest rate.

Explanations:

Q. 1: While the slope of line depicting sales of B is same for all the years (it is a straight line), the percentage growth in sales is not the same in each year. Reading the values from the Y-axis, the sales in successive years can be found as 10, 20, 30, 40 and 50. The percentage growth in successive years from 2000 will hence be $\frac{10}{10}, \frac{10}{20}, \frac{10}{30}$ and $\frac{10}{40}$ i.e. 100%, 50%, 33.33% and 25%.

Thus, same slope of each segment does not indicate same growth rate. Infact, if the slope is same, and the line is increasing, it always implies a decreasing growth rate. Read on the next explanation and then the box that follows to understand what does slope refer to.

Q. 2: Even though the slope of line segment depicting sales of A and that of B have the same slope over the years 1999 to 2000, the growth rate of sales of A in 2000 and that of sales of B in 2000 is not the same.

Sales of A grew from 10 to 20, a growth rate of 100%. Sales of B grew from 20 to 30, a growth rate of 50%.

Same is the scenario for the years 2000 to 2001. Sales of A grew from 20 to 30, a growth rate of 50% and sales of B grew from 30 to 40, a growth rate of 33.33%. Thus, even though these two respective line segments have the same slope, the growth rates are different

What does slope refer to?

We know that slope of a line is $\frac{\text{change in Y}}{\text{change in X}}$

Change is Y is the increase in the underlying value and this is exactly what the numerator should be to find the growth rate.

But what is change in X in the graph? The X-axis of the graph is successive time periods. Thus, the change in X, from one reading to the next reading is just 'one time period'. But this is not what is needed to find the growth rate. To find the growth rate we need to divide it by the 'initial' underlying value. Hence the above interpretations are not correct.

Observation 1: Same slope simply means that the actual growth (in amount and not percentage) from one reading to the next reading is the same. Thus, since line depicting sales of B was straight i.e. had same slope in each time period, hence the growth in sales (in Rs. million) was the same in each time period, a increase of 10 in each successive year as noticed earlier.

Observation 2: For an increasing straight line, the increase is contant, but the base is continuously increasing. Hence the growth rate in successive years has to be lesser and lesser.

Observation 3: For two line segments of the same slope but placed differently in the graph e.g. line segment depicting sales of A from 1999 to 2000 and the line segment depicting sales of B from 1999 to 2000, having the same slope means the increase in sales from 1999 to 2000 is the same for both the companies. But again since the line segment of A is placed higher than the line segment of B, it means the base in the case of A is more than that in case of B. Hence the growth rate of A is lesser than the growth rate of B in the period.

Thus, among two line segments having same slope, the one which is placed 'higher' on the graph will have a lower growth rate.

In a nutshell, to have a higher growth rate, as the position of the line segment keeps getting higher and higher up in the graph, the inclination of the line has to get more and more steeper. Thus, only when there is a 'marked' increase in steepness of line segments, would the growth rate be more.

Q. 3: Look at the line depicting sales of A. It is a straight line from 1999 to 2000 to 2001. But from 2001 to 2002, it is steeper. As read in the box, don't be too hasty to judge that the growth rate over the years 2001-2002 is the highest. For this to happen the 'steepness' should be 'markedly higher'. For such 'marginal increases' in steepness, it is best to check with values.

Sales of A in successive years can be found as 20, 30, 40, 55. Thus, the growth rates are $\frac{10}{20}, \frac{10}{30}, \frac{15}{40}$ i.e. 50%, 33.33% and 37.5%. Thus, we see that highest sales growth rate is over the years 1999-2000.

So should we trust the slope as a measure of growth rate or not?

Visually one should check for a steeper line segment and narrow down your choices as most often it will refer to the highest growth rate. But keep in mind the following points also:

1. Always also take into account any line segment that appears far too 'low down' in the entire line graph. This could also be the period with greatest growth because the base is very less in this case.

2. Look at the entire growth of the line and the range of values. In this case the reading of the line increased from 10 to 40, an increase of 400%. These are very low numbers and the increase is also very very much. If the increase is not so much, say the entire range is just from 150 to 180. With such range, the steeper line will usually be the one with the highest growth rate.

3. And anyways, to compare growth rate in line segments of same slope, you can always use the point that same slope but lower base would mean higher growth rate.

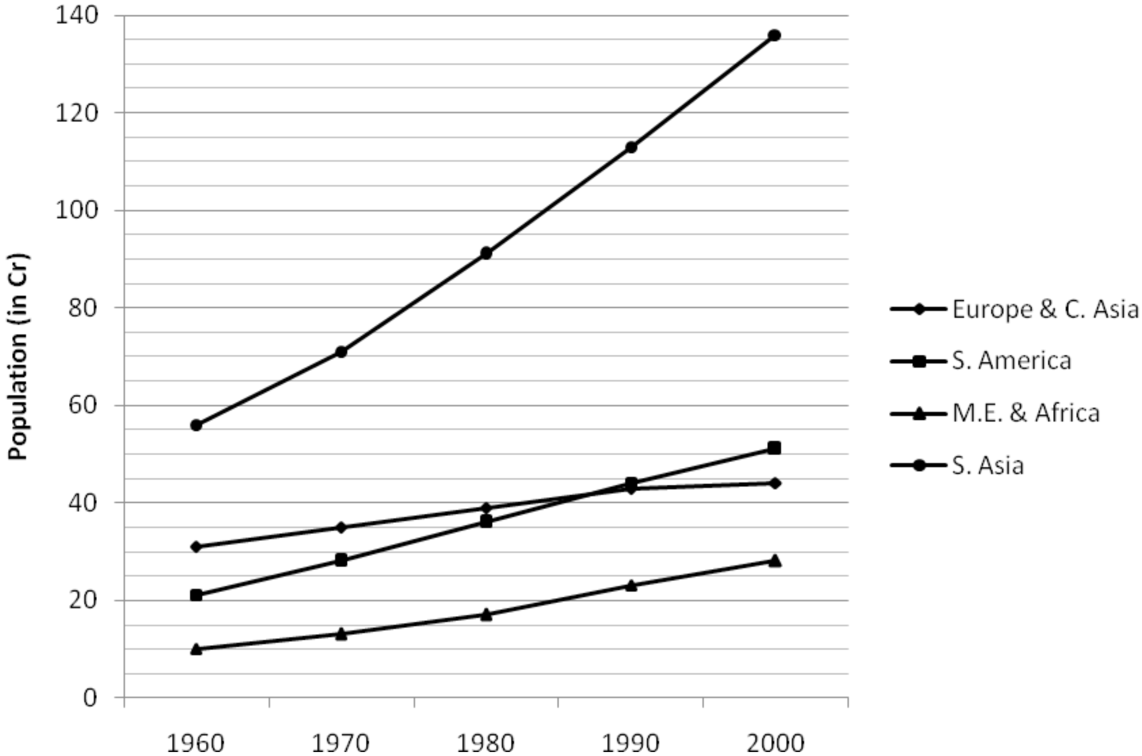
Q 4: The sales of C in successive years is 45, 35, 20, 15 and 8. Thus the growth rates are –18.18%, –42.84%, –25% and –46.66%.

Thus the rate of greatest decrease is maximum in the year 2002-2003.

Next an exercise for you to practice.

Exercise 5:

The following line graph shows the population (in cr) of four major regions of the world from 1960 to 2000.



1. In which decade did the population of S. Asia increase by the greatest rate?

   a. 1960-70 b. 1970-80 c. 1980-90 d. 1990-2000

2. From 1950 to 2000, by what percentage did the population of M.E. & Africa increase?

   a. 180% b. 280% c. 80% d. Cannot be determined

3. For how many regions is the growth rate of population continuously decreasing in successive decades?

   a. None b. One c. Two d. Three

4. For which region and decade was the growth rate of population the highest across all the four region between the time period 1960 to 2000?

   a. M.E. & Africa, 1970-1980 b. M.E. & Africa, 1980-1990

   c. S. Asia, 1960-1970 d. S. Asia, 1970-1980

5. Which region's population grew by the maximum rate over the period 1960 to 2000?

   a. Europe & C. Asia b. S. America

   c. M.E. & Africa d. S. Asia

6. Which region's population grew the most over the period 1960 to 2000?

   a. Europe & C. Asia b. S. America

   c. M.E. & Africa d. S. Asia

7. In the decade 1060-70, which region grew at the highest rate?

   a. Europe & C. Asia b. S. America

   c. M.E. & Africa d. S. Asia

8. What was the average annual growth rate of population of Europe & C. Asia in the period 1980-1990?

   a. 10.3% b. 7.7% c. 4.2% d. 1%

Calculation Techniques

While solving the above exercises, we would very often have to perform a division to find the growth rate e.g. $\frac{200}{680}\times100$, $\frac{182}{128}\times100$.

In this topic we will learn few techniques to find the answer without getting frustrated with the calculations. And our "brahma-astra" will be the option choices. Thus, the following text is not meant for any scenario, but specifically about management entrance tests where options choices are given. Option choices are our best allies, and if you do not take help of them, you would be committing an "unpardonable sin". The moment you have to do a calculation, you HAVE to notice the option choices first and only then proceed. If you do so, never will you be frustrated.

First we will learn the techniques and then apply them to the Exercise 1.

Let's say that we need to do the division $\frac{654}{852}$ and the option choices are

a. 65% b. 71% c. 76% d. 80% e. 82%

Say, $\frac{654}{852} = x\,\%$, then 654 = x % of 852.

Thus the percent that we are finding, x%, is that of the denominator, in this case of 852.

Also one should immediately bear in mind that $10\%\text{ of }852 \cong 85$ and 1% of 852 = 8.5

From the options one can easily deduce the range of x as 65% to 81%.

We first find a percentage of 852 that is easy to find and is also in the range of the option choices. In this case we could have found 70% or 80% of 852. There are many more percentages within the range that can be found easily. Just start with any one value.

$70\%\ of\ 852 \cong 85\times7 \cong 595$

But since x% of 852 has to be 654, x has to be more than 70%. How much more?

654 = 595 + 59. And since 1% of 852 = 8.5, 59 will be around 7% of 852.

Thus 654 = 70% + 7% i.e. approximately 77% of 852.

Among the options, the answer can now easily be identified as 76%.

E.g. : $\frac{2512}{1089} = ?$

a. 2.15 b. 2.30 c. 2.45 d. 2.60

Since all options are more than 2, hence we start with finding 2 × 1089 = 2178.

2512 = 2178 + 334

Since $0.1 \times 1089 \cong 109$, hence $334 \cong 0.3 \times 1089$

Thus, 2512 = (2 + 0.3) × 1089 and the answer will be 2.3 i.e. option (b)

Approximation Technique

The above method does not rely or does not use approximation techniques. The method can be used to reach any desired level of accuracy (depending on how close the options are).

There are also approximation techniques, where we change one of the numerator or denominator to a more manageable number.

Approximate 1089 as 1100. By actual division we can find that $\frac{2512}{1100} = 2.28$.

Since we have increased the denominator, the answer found will be lesser than the actual answer. Thus, the correct answer has to be more than 2.28. The answer has to be 2.30, among the choices.

But approximation techniques could sometimes need a lot of creativity and further few students may have a doubt even after approximation (e.g. in the above questions while the answer is greater than 2.28, why is it only 2.3 and not 2.45?) So we do not recommend approximation techniques to someone not strong in calculation.

(The answer cannot be 2.45 because in approximating 1089 as 1100, an error of approximately $\frac{11}{1089} \cong 1\%$ is introduced and thus 2.45 cannot be the answer as it is almost 10% higher than 2.28)

If you have to do approximation, it is advisable to approximate just one number and NOT more than one. With this we can safely know whether the actual answer is more than or less than the approximated answer.

The following examples are based on calculation that you would have encountered in Exercise 1. While following the examples do relate it to the amount of calculation you have done while solving the exercise. Notice how hardly any calculations are involved.

Soln. 1: We need to find $\frac{200}{680}$ and options are

a. 7.5% b. 10% c. 22.5% d. 30%

10% of 680 = 68.

What multiple of 68 would be around to 200? Its almost 3 × 68, hence answer is straight 30%

Soln. 2: Cement market has grown from Rs. 1990 (lacs) to Rs. 2720 (lacs), we need to find $\frac{730}{1990}$ and the options are

a. 9% b. 12% c. 30% d. 36% e. 40%

10% of 1990 ≅ 200.

What multiple of 200 will be around 730? It will be a little more than 3.5 × 200.

Thus, 730 is more than 35% of 1990. And the answer among the given options should be clearly 36%.

(If one needs more accuracy: 35% of 1990 ≅ 700. We have to account for another 30. Since 1% of 1990 ≅ 20, 30 will be around 1.5 percent of 1990. Thus a more accurate answer would be 36.5%)

Soln. 3: We need to find by what percent is 1350 more than 1160 i.e. the ratio $\frac{190}{1160}$ and the options are

a. 14% b. 16% c. 18% d. 20%

10% of 1160 = 116

190 = 116 + 74.

1% of 1160 = 11.6

What multiple of 11.6 will be around 74? It is more than 6 × 11.6 and less than 7 × 11.6.

Thus answer has to be more than 10% + 6% and less than 10% + 7% and among the options the choice would be 16%.

Soln. 4: The percentage change in average sales will be the same as the percentage in total sales. Total sales of C and A is 3100 and 1280 respectively. Thus, we have to find $\frac{1820}{1280}$ and the options are

a. 42% b. 67.6% c. 114% d. 142%

100% of 1280 = 1280

Numerator is 540 more than 100% of 1280.

10% of 1280 = 128.

What multiple of 128 would approximately be 540? Just a little more than 4.

Thus 540 will be a little more than 40% of 1280.

And our answer is 142%.

Soln. 5: We need to add 15% of 1465 to itself and the options are .

a. 1685 b. 1665 c. 1705 d. Cannot be determined

10% of 1465 = 146 and hence 5% of 1465 will be half of it i.e. ≅73

Thus sales = 1465 + 146 + 73 = 1684

Soln. 6: From theory of percentages we would know that $37.5\% = \frac{3}{8}$ and if a number has to be increased by 37.5%, it has multiplied by $\frac{11}{8}$.

Thus, $\text{Sales}_{2002} \times \frac{11}{8} = 680 \Rightarrow$ Sales in 2002 ≅ 8 × 61.8 =480 + 8 + 6.4 = 494.4

Soln. 7: We need to find by what percent is 375 less than 680 i.e. the ratio $\frac{305}{680}$ and the options are ......

a. 45%   b. 55%   c. 67.5%   d. 81%

10% of 680 = 68

What multiple of 68 would be around 305? It would be more than 4 times and less than 5 times.

Thus answer from the option choices has to be 45%

Soln 8: First let us find the simple annual growth rate of increase from 300 to 375 over two years.

It would be $\frac{75}{300} \times 100\% \times \frac{1}{2} = 12.5\%$

Since CAGR < SAGR, the answer from the options choices will be 12% (It can't be 10% because over 2 years period, the difference in CAGR and SAGR would be very minimal)

Alternately one has to find $\sqrt{\frac{375}{300}} = \sqrt{\frac{5}{4}} = \sqrt{1.25}$, which would require approximation techniques.

When options are very close

In cases where the option choices are absolutely close (even in they differ only in the second digit after decimal point), even then one can use the options choices very effectively. Such questions become all the more easier because of the close option choices! Surprised? How? Because you already know the answer has to be in the narrow range given.

E.g. 13: $\frac{1112}{2089} = ?$

a. 0.50   b. 0.51   c. 0.52   d. 0.53   e. 0.54

Since the answer is close to 0.5, finding 0.5 × 2089 = $\frac{2089}{2}$ = 1044.5

Next, 1112 = 1044.5 + 67.5

0.01 × 2089 = 20.89. What multiple of 20.89 will be approximately 67.5? It has to be a multiple between 3 and 4 (and closer to 3). Thus, 67.5 ≅ 0.03 × 2089

And our answer will be 0.53

E.g. 14: $\frac{26140}{2948} = ?$

a. 8.86 b. 8.87 c. 8.88 d. 8.89

First aim should be to find 2948 × 8.8. Since 2948 × 8 = 23584

2948 × 8.8 = 23584 + 2358.4 = 25942.4

The numerator, 26140 = 25942.4 + 197.6

Next, 0.01 × 2948 = 29.48. What multiple of 29.48 will be approximately 197.8? It has to be a multiple between 6 and 7 (and closer to 7). Thus, 197.6 ≅ 0.07 × 2948.

And the required answer would be 8.8 + 0.07 = 8.87

The following is an exercise to get yourself to master the above techniques.

Calculation Exercise:

The idea is to finish this exercise within 20 minutes with more than 80% accuracy i.e. 16 corrects. So do measure your time & performance.

Start Time: ____________

1. $\frac{421}{730} = $ ____

   a. 0.42 b. 0.47 c. 0.52 d. 0.57

2. $\frac{2045}{8252} = $ ____

   a. 0.21 b. 0.25 c. 0.29 d. 0.34

3. $\frac{1006}{1279} = $ ____

   a. 0.72 b. 0.75 c. 0.78 d. 0.81

4. $\frac{259}{1953} = $ ____

   a. 0.123 b. 0.132 c. 0.119 d. 0.125

5. $\frac{52}{715} = $ ____

   a. 0.72 b. 0.072 c. 0.075 d. 0.078

6. $\frac{250}{117} = \_\_\_\_$

a. 2.1 b. 2.13 c. 2.18 d. 2.21

7. $\frac{2852}{1729} = \_\_\_\_$

a. 1.65 b. 1.62 c. 1.68 d. 1.58

8. $\frac{8522}{363} = \_\_\_\_$

a. 21.5 b. 20.5 c. 23.5 d. 22.5

9. $\frac{1835}{1259} = \_\_\_\_$

a. 1.41 b. 1.36 c. 1.46 d. 1.51

10. $\frac{12011}{1011} = \_\_\_\_$

a. 11.08 b. 11.38 c. 11.68 d. 11.88

11. $\frac{880}{712} = \_\_\_\_$

a. 1.23 b. 1.32 c. 1.43 d. 1.54

12. $\frac{103}{218} = \_\_\_\_\%$

a. 44.4% b. 45.5% c. 46.3% d. 47.2%

13. $\frac{458}{634} = \_\_\_\_\%$

a. 63.5% b. 66.3% c. 69.7% d. 72.2%

14. $\frac{398}{551} = \_\_\_\_\%$

a. 69.5% b. 72.2% c. 75.8% d. 78.5%

15. $\frac{323}{765} = \_\_\_\_\%$

a. 37.5% b. 39.4% c. 42.2% d. 45%

16. $\frac{3841}{4831} = \_\_\_\_\%$

a. 79.5% b. 82.8% c. 85.7% d. 89.5%

17. $\frac{4010}{5711} = \_\_\_\_\%$

a. 64.8% b. 67.5% c. 70.2% d. 74.3%

18. $\frac{6092}{9437}$ = ____%

a. 56.8% b. 59.2% c. 61.3% d. 64.5%

19. $\frac{4421}{5644}$ = ____%

a. 78.3% b. 80.5% c. 83.7% d. 85.9%

20. $\frac{1310}{1828}$ = ____%

a. 68.8% b. 71.6% c. 74.2% d. 77.7%

End Time: ____________

Calculation techniques to compare Growth Rates (or any ratios)

Questions are equally divided between finding a growth rate and comparing few growth rates to find the greatest (or smallest) growth rate. Say, we need to compare growth rates in 4 different years and find the greatest growth rate. For this question, if we are going to find out the growth rates for each of the 4 years, our calculation job is multiplied four times. Rather than calculate each of the individual 4 growth rates and then compare, we should try to compare the growth rates without finding any of them individually, as the following example shows

Year:	2005	2006	2007	2008	2009
Sales:	3485	3796	4065	4450	4980

Which of the years showed the maximum percentage growth over the previous year?

The increase in 2006 can be found as 15 + 296 = 311 and thus, the percentage growth will be $\frac{311}{3485}$.

The increase in 2007 can be found as 4 + 265 = 269 and thus, the percentage growth will be $\frac{269}{3796}$.

The increase in 2008 can be found as 35 + 350 = 385 and thus, the percentage growth will be $\frac{385}{4065}$.

The increase in 2009 can be found as 50 + 380 = 430 and thus, the percentage growth will be $\frac{430}{4450}$.

Thus, we have to compare which of $\frac{311}{3485}, \frac{269}{3796}, \frac{385}{4065}, \frac{430}{4450}$ is the greatest.

Mere observation will rule out the second one since the numerator is less than that of the first ratio whereas the denominator is more than that of first. Cancel out the ratio in your work-sheets and now we have to only focus on three, $\frac{311}{3485}, \cancel{\frac{269}{3796}}, \frac{385}{4065}, \frac{430}{4450}$

Checking in what range would each ratio be, we can notice that each is just less than 10%. Thus, the three can be thought of as:

10% of 3485 is approx 348 and the numerator, 311, is 37 less than 348. Similarly for the other two ratios and we get the three ratios as:

10% less $\frac{37}{3485}$; 10% less $\frac{21}{4065}$; and 10% less $\frac{15}{4450}$.

Since we want the maximum growth rate, the part that we are subtracting from 10% has to be the least. And it is obvious that $\frac{15}{4450}$ is the least since its numerator is the least and denominator is greatest. Thus, the fourth growth rate i.e. in year 2009 is the greatest.

Let's see some more examples of comparing data.

E.g. : The following data gives India's exports to and imports from UAE for the period 2004-05 to 2008-09. All values are in Rs. bn.

2004-05		2005-06		2006-07		2007-08		2008-09	
Export	Import	Export	Import	Export	Import	Export	Import	Export	Import
330.15	208.53	380.38	192.77	544.45	391.75	629.15	542.33	1102.3	1059.2

1. Which year showed the least percentage growth in exports to UAE?
   a. 2005-06 b. 2006-07 c. 2007-08 d. 2008-09
2. Which year showed the maximum growth rate in imports from UAE?
   a. 2005-06 b. 2006-07 c. 2007-08 d. 2008-09
3. For which year was the ratio of imports from UAE to the exports to UAE the highest?
   a. 2005-06 b. 2006-07 c. 2007-08 d. 2008-09

Solutions:

Q. 1: The exports in successive years were approximately 330, 380, 544, 629 and 1102.

A visual check is enough to realise that the least growth will be from 330 to 380 or from 544 to 629. The other two cases, the growth rate is very high (from 380 to 544 i.e. an addition of 160, roughly between 40% and 50%; and from 629 to 1102 is going to be even higher in the vicinity of 80% types)

Thus, we are just left to compare $\frac{50}{330}$ and $\frac{85}{544}$

Method 1: Approximate the growth rates

Finding the first ratio: 10% of 330 is 33; 5% of 330 is 16.2.

Thus, 15% will be 33 + 16.2 = 49.2. Hence the first ratio is almost 15%

Finding the second ratio: 10% of 544 is 54.4; 5% of 544 is 27.2.

Thus, 15% will be 54.4 + 27.2 = 81.6. Hence the second ratio is slightly more than 15%.

So the least growth rate is from 330 to 380 i.e. in the year 2005-06, option (a)

Method 2: Compare the growth rates in the numerators with the growth rate in the denominators

Going from the first ratio to the second ratio, the numerator increases from 50 to 85 i.e. a percentage increase of 70%; and the denominator increases from 330 to roughly 550 i.e. a ratio of 3 : 5 i.e. a percentage increase of 66.66%.

Thus, going from first ratio to second, the numerator increases faster (higher percentage) than the denominator. This will make the second ratio more than the first. Thus, 50/330 is the smaller among the two.

Q. 2: The imports in successive years were approximately 208, 193, 392, 542 and 1059. And we need to find the largest growth rate between two successive values.

Obviously from 208 to 193 is a decrease and so is out of contention.

193 to 392 is a growth of 7 + 192 = 199 and growth rate is $\frac{199}{193}$, which is more than 100%.

So we have bench-mark figure to compare other growth rate and a visual check will confirms that neither did 392 double in the next year and nor did 542. Thus, the growth rate found, from 193 to 392 is the largest.

Thus, the largest growth rate is in the year 2006-07, option (b)

Q. 3: While this question is not about a growth rate, yet the spirit of the question still requires us to compare five ratios, viz. $\frac{209}{330}, \frac{193}{380}, \frac{392}{544}, \frac{542}{629}, \frac{1059}{1102}$ (We can always start with round figures and only if two ratios turn out to be very very close, can we use the actual decimal values). And we need to find the highest ratio among these five.

Starting with the first two ratios, $\frac{209}{330}, \frac{193}{380}$, the second ratio is almost half, whereas the first ratio is more than half. Thus, the second ratio is ruled out as the answer.

Glancing at the other ratios and checking if any ratio is far far greater than half, we can immediately zero down on $\frac{542}{629}$ & $\frac{1059}{1102}$.

To compare this ratio, we have two methods as learnt in the Q. 1. To mention it again, we can approximate each ratio to a percentage OR we can compare the growth rate in the numerators and in the denominators. Method 2 seems to be easier, so let us use this first:

From the first ratio to the second, both the numerator and denominator increased to a little less than twice, so we will have to be more precise. The numerator increased to 'twice less $\frac{25}{542}$' i.e. numerator increased 1.95 times (this should be done orally because 25/500 is 0.05). Whereas the denominator increased to 'twice less $\frac{156}{629}$' i.e. the denominator increased '2 – 0.25' time i.e. 1.75 times.

From $\frac{542}{629}$ to $\frac{1059}{1102}$, the numerator increases faster than denominator and hence the ratio has to increase. Thus, $\frac{1059}{1102}$ will be the greater among the two.

Alternative way:

Looking at the two ratios $\frac{542}{629}$ & $\frac{1059}{1102}$, it is far easier to divide by 11. Thus, the second of these ratios, by actual division by 11, can be found as 0.962

Now we have a bench-mark to compare the first ratio with. Finding 0.9 of 629, or rather 0.9 of 630, we get the value as 567. Thus, $\frac{542}{629}$ will be less than 0.9.

In short, the above solutions try to tell you that there is no one single approach that one should follow. But it also tells that one should first check visually and eliminate two or three options. And when comparison has to be done between two close options, one can see if any of the ratio is easy to calculate by actual division; if the ratio can be approximated to a value by finding 10% of the denominator; if the two numerators and the two denominators can be compared. And then one can proceed. In any case, you have to be very nimble footed and adapt your approach. Next, we will take a few comparison of ratios given directly and then follow up with a DI set that have questions about comparing growth rates or ratios.

Calculation Exercise (continued):

21. Which of the following is the largest?

a. $\frac{419}{471}$ b. $\frac{498}{533}$ c. $\frac{488}{598}$ d. $\frac{455}{519}$

22. Which of the following is the largest?

a. $\frac{200}{245}$ b. $\frac{210}{255}$ c. $\frac{231}{283}$ d. $\frac{249}{282}$

23. Which of the following is the smallest?

a. $\frac{26}{36}$ b. $\frac{50}{64}$ c. $\frac{27}{41}$ d. $\frac{70}{92}$

24. Which of the following is the largest?

a. $\frac{274.8}{557.7}$ b. $\frac{299.3}{584.1}$ c. $\frac{240.3}{582.4}$ d. $\frac{280.7}{591.7}$

25. Which of the following is the smallest?

a. $\frac{161.1}{292.5}$ b. $\frac{123.9}{213}$ c. $\frac{111.5}{237}$ d. $\frac{116.1}{226.4}$

26. Which of the following is the smallest?

a. $\frac{78}{67}$ b. $\frac{74}{63}$ c. $\frac{75}{62}$ d. $\frac{80}{68}$

27. Which of the following is the second smallest ratio?

a. $\frac{390}{232}$ b. $\frac{426}{272}$ c. $\frac{367}{186}$ d. $\frac{371}{259}$

28. Which of the following is the smallest?

a. $\frac{2461}{7528}$ b. $\frac{246}{752}$ c. $\frac{24}{75}$ d. $\frac{2}{7}$

29. Arrange the three in increasing order: I. $\frac{0.96}{1.63}$ II. $\frac{0.79}{1.31}$ III. $\frac{0.84}{1.58}$

a. III, II, I b. III, I, II c. I, III, II d. II, I, III

30. Arrange the three in decreasing order: I. $\frac{3045}{3245}$ II. $\frac{3427}{4154}$ III. $\frac{1643}{2034}$

a. I, II, III b. III, II, I c. I, III, II d. II, III, I

Exercise 6:

The following table gives the data about India's imports from and exports to the 5 largest trading partners of India for the period 2004-05 to 2008-09. Read the table carefully and answer the questions that follow. All values are in Rs. bn.

Country	2004-05		2005-06		2006-07		2007-08		2008-09	
	Export	Import	Export	Import	Export	Import	Export	Import	Export	Import
UAE	330.15	208.53	380.38	192.77	544.45	391.75	629.15	542.33	1102.3	1059.2
China	252.3	319	299.2	481.2	375.3	790.1	436	1091.1	426.6	1476.1
USA	618.5	314.6	768.2	418.6	853.6	531.1	833.9	846.3	954.6	848.2
Saudi Arabia	63.4	58.5	80.1	72.3	117.1	605.6	149.2	781.1	229.4	897.5
Germany	127	180.4	158.8	266.7	180.1	341.5	206	397.4	291.9	549.2

1. In which year did the exports to China show the largest percentage growth?

   a. 2005-06 b. 2006-07 c. 2007-08 d. 2008-09

2. In which year did the imports from Germany show the largest percentage increase?

   a. 2005-06 b. 2006-07 c. 2007-08 d. 2008-09

3. Exports to UAE in the year 2007-08 was how much percent more than the imports from UAE in the same year?

   a. 12% b. 14% c. 16% d. 18%

4. By what approximate average annual percentage did imports from Germany grow over the period 2004-05 to 2008-09?

   a. 50% b. 32% c. 40% d. 200%

5. In the year 2004-05, which country had the highest ratio of exports to imports?

   a. UAE b. China c. USA d. Saudi Arabia

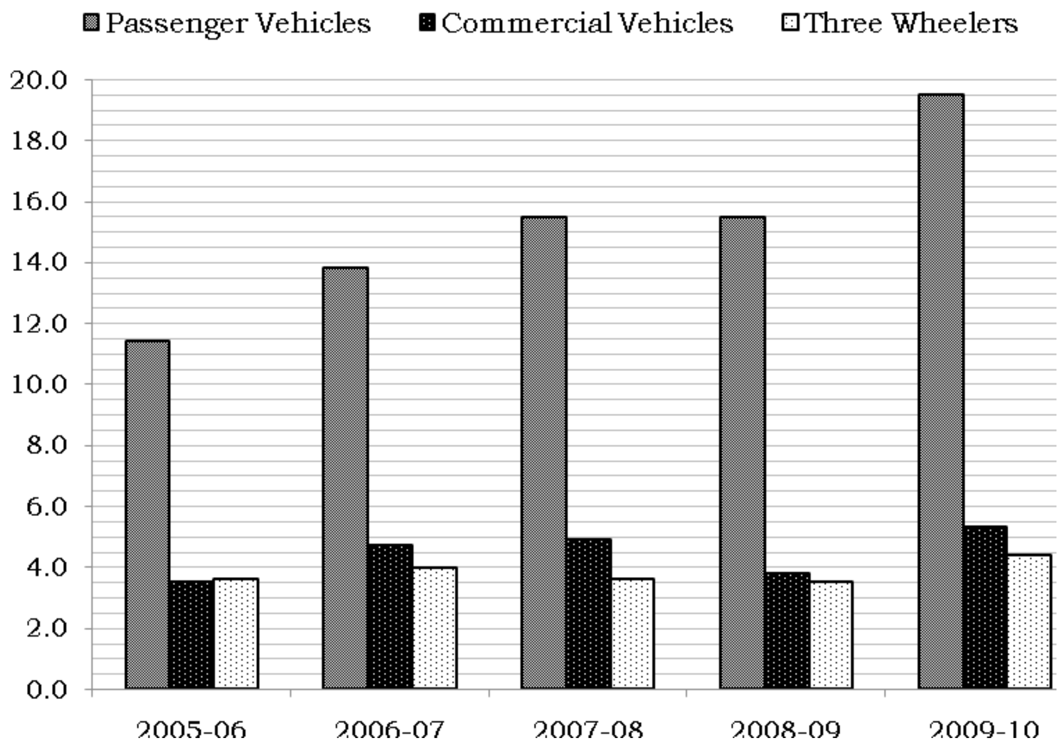
6. Which country showed the least average annual growth rate in imports over the period 2004-05 to 2008-09?

   a. Germany b. Saudi Arabia c. USA d. China

The same questions on growth rate can also be asked with graphs being given instead of tables. The only difference is that now you will have to read out the data from the graphs and not from the tables. To start with, the following exercise gives data very similar to the data we encountered in above example and exercise, except that it is in the form of a simple bar graph, reading of which will be familiar to most. Also use the knowledge learnt about linking growth rates and slope of lines to compare growth rates.

Exercise 7:

The following bar graph gives the number of vehicles sold in India for three categories of vehicles – passenger vehicles, commercial vehicles and three wheelers, over the years 2005-06 to 2009-10. Read the graph carefully and then answer the questions that follow. All values in the graph are in lacs.



1. From 2005-06 to 2009-10, which category of vehicles showed the highest compounded annual growth rate?

   a. Passenger vehicles b. Commercial vehicles

   c. Three Wheelers d. Cannot be determined

2. In which year did commercial vehicles register the largest growth rate?

   a. 2006-07 b. 2007-08 c. 2008-09 d. 2009-10

3. In the year 2006-07, which category of vehicles registered the least growth rate?

   a. Passenger vehicles b. Commercial vehicles

   c. Three Wheelers d. Cannot be determined

4. What was the growth rate in the total number of vehicles, across all the three categories, sold in the year 2008-09 over those sold in the year 2006-07?

   a. 0.13% b. 1.3% c. 13.3% d. 0.013%

5. In 2006-07, commercial vehicles accounted for what percent of the total sales of the three categories of the vehicles?

   a. 19.5% b. 20.5% c. 21.5% d. 22.5%

6. By what percent did the sales of the three categories of vehicles grow from 2007-08 to 2008-09?

   a. 7.5% increase b. 5% increase c. 7.5% decrease d. 5% decrease

Market Share, Basic Pie Charts, Value & Volume

Market Share is another term that one will come across very often in DI. Though the name itself is very self explanatory, for those who are not clear about its meaning a very basic primer on the associated terms follows.

Market

'Market' refers to the aggregation (sum) of all sellers of a particular product. For e.g. the market for automobiles would refer to the total of all automobile manufactures like Maruti, Hyundai, Honda, Ford, Tata, Bajaj, Hero, Volvo, etc. Usually there will be some restriction like 'Indian Car Market'. So here we are talking of only car manufacturers and that too only in India. So this 'market' would refer to the total of all the relevant players in the sector/ geographical area.

Market Share of an Individual Player

Share as the meaning suggests is that part of the total market that is made up by the individual player. And it is usually expressed in percentage terms, e.g. 10% of the market. E.g. If total sales of all the cars in India was 20,000 and of these 20,000, sales of Maruti was 8,000, then Maruti's share of the market would have been $\frac{8,000}{20,000} \times 100 = 40\%$

Thus, market share of individual player = $\frac{\text{Sales of that individual player}}{\text{Total sales of all the players}} \times 100$

Market & Share could be in Value or Volume terms

The total market could be expressed in different terms. E.g. in the example of the car market, we could have said that altogether 20,000 cars were sold. Here we are talking of number of cars and this is called Volume sales. Alternately we could also have said that the total value of all cars sold is Rs. 1,500 crores. In this case, we are talking of the market in Value terms.

Accordingly, based on whether we are talking about volume or value, the market share could also be Volume share or Value Share. More on this and their interlinking in the next topic.

Needless to add, the sum of market shares of all the individual players has to be 100%, because all of them together account for the entire market i.e. for 100% of the market. Thus, if there are only three players in a particular market and the market share of two of them is 20% and 30%, the rest of the market, 100% – 20% – 30% = 50% has to be the market share of the third player.

The above understanding should be enough to fill in the following table:

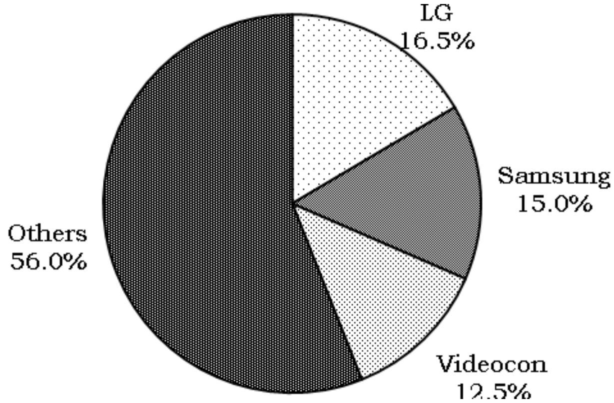
Exercise 8:

Fill in all the six blank cells in the following table.

Sales of CTVs in India in 2005		
Manufacturer	Sales (Rs. Mn)	Market Share
LG		16.50%
Samsung	180	15%
Videocon	150	
Others		
Total Market		

Pie Charts

Pie Charts are the most convenient form to represent any 'Share'. The name, pie, itself suggests an entire pie that is shared among many players. Again most of you would be familiar, that the entire pie, denoted by a circle, refers to the entire market. And this circle is then divided into sectors, each sector representing the individual share of the players. The depiction of market share for the above CTV market and sales values is:



Larger the market share of an individual player, larger will be his sector. Infact, the size of the sector is directly proportional to the market share. Since 'Others' have more than half the market, more than 50%, their sector is also more than half the circle. Thus, the pie chart is a nice visual comparison between all players.

Angle of the sector

100% of the market refers to the entire circle i.e. an angle of 360 degrees at the center.

Since the size of the sector, usually expressed in measure of degree of angle formed at the center of the circle, is directly proportional to the market share, hence we could conclude that

1% market share $\Rightarrow \frac{360}{100}$ i.e. 3.6 degrees at the center.

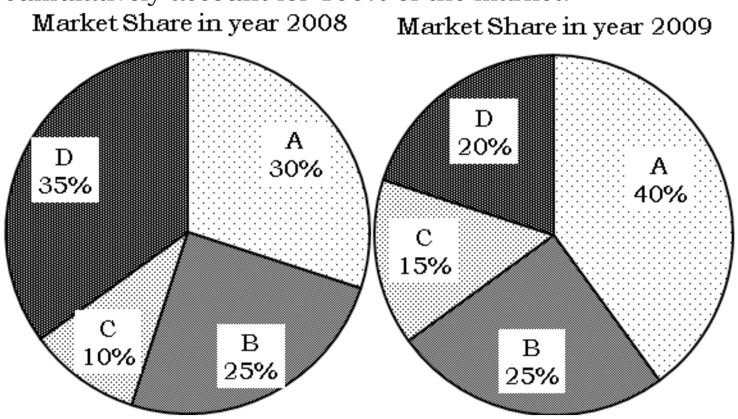
Hence in the above example, the sector depicting Samsung's share would have 15 × 3.6 = 54 degrees at the center, whereas the sector representing Others would have 56 ×3.6 = 201.6 degrees at the center (more than half, 180 degrees)

Two Pies depicting shares for two years

The most simplest type of data given in pie charts is two pies denoting the market share of the players in two different years, as given below. We will take the following as an exercise such that the understanding of such pie charts is perfect and not spoon-fed or mugged up. The exercise has subjective questions which you should answer before looking at the explanations. The numbers used are very easy ones so that you don't get lost in calculations but rather focus on the understanding of the concept.

Exercise 9:

The following graph depicts the market shares of four companies, A, B, C and D in two years, 2008 and 2009. The graph to the left is for year 2008 and the graph to the right is for year 2009. These four companies are the only players in the market and hence cumulatively account for 100% of the market.



Please not that with the above data, we only know the market shares. We do not know the actual value of the total market and hence cannot ascertain the actual sales value of companies, A, B, C and D. We can only say that in 2008, D's sales was 35% of the total market sales and it was the highest among the four companies and so on.

1. Since A's share of the pie has increased from 2008 to 2009, does it mean that sales of A has increased from 2008 to 2009? YES / NO

1a. If your answer is NO, find the condition (related to total market sales) under which the sales of company A would have

Case i: Increased   Case ii: Remained stagnant   Case iii: Decreased

2. Since D's share of the pie has decreased from 2008 to 2009, does it mean that sales of D has decreased from 2008 to 2009? YES / NO

2a. If your answer is NO, find the condition (related to total market sales) under which the sales of company D would have

Case i: Increased   Case ii: Remained stagnant   Case iii: Decreased

3. Since B's share of the pie has remained constant from 2008 to 2009, does it mean that sales of B has remained constant from 2008 to 2009? YES / NO

3a. If your answer is NO, find the condition (related to total market sales) under which the sales of company B would have

Case i: Increased   Case ii: Remained stagnant   Case iii: Decreased

4. Can the growth rate of sales of the individual companies be found out? YES / NO

4a. If the entire market has grown by 10%, now is it possible to find the growth rate of the individual companies? YES / NO

4b. If your answer to 4b is YES, find the individual growth rates of sales for all the four companies.

Further can you find the relation between the growth rate of market, market shares of an individual company in the two years and the growth rate of sales of that individual company?

5. Can we compare the growth rate of sales of individual companies (without knowing any other data except that given in the pie-charts)? By comparing we mean can we rank the companies in increasing (or decreasing) order of their growth rate of sales? YES / NO

If your answer is YES, please rank the companies in increasing order of the growth rate of their sales.

6. If the sales of C has increased by 10% from 2008 to 2009, is possible to find the growth rate of the market? And also of the growth rate of sales of all other companies? YES / NO

If your answer is YES, please find the growth rate of the total market and also growth rate of sales of A, C and D.

Solutions and Fundas:

The values given in the pie are only share of the pie. Thus one may have a certain share of pie 1 and a larger (or smaller) share of pie 2. But this does not necessarily that one has got a bigger piece (or smaller piece).

What would you want – one-half of cake 1 or one-fourth of cake 2 (assuming you want as much cake as possible)? One cannot make a choice unless the two cakes are compared. If cake 1 is only 1 kg, one-half of that cake is only 500 gm. And if cake 2 is 4 kgs, even though we have a lesser share, one-fourth, yet in absolute terms it is more than earlier, it is 1 kg.

This should be enough to conclude the answers as NO for Q. 1, 2, and 3. Continuing the same thoughts, in an increasing market, even if I am loosing share, my sales could go on increasing because my share is getting lower, but the entire cake is getting larger (yes, it has to do with the rate at which share is decreasing and rate at which cake is increasing, but you get the idea, right?).

When would two pieces of two different cakes be equal?

Consider cake 1 to be C_1 and its share to be S_1, cake 2 to be C_2 and its share to be S_2. The two pieces would be equal when $S_1 \times C_1 = S_2 \times C_2$ i.e. when $\frac{C_2}{C_1} = \frac{S_1}{S_2}$ i.e. when the cakes are inversely proportionally to the shares.

Thus, considering question 1b, the market share of A, in 2008 and 2009, is in the ratio 30 : 40 i.e. 3 : 4. Thus, then IF the entire pie i.e. the entire market is in the ratio 4 : 3, the pieces of the cake (sales of A in this case) will be equal. Thus, if the market has decreased by 25%, even though share of A has increased from 30% to 40%, sales of A would be the same. If the market has decreased by anything more than 25%, A's sales would have decreased (and conversely if market had decreased by less than 25%, A's sales would have increased).

Using the same funda, one can answer Q. 2b and 3b.

Infact, Q3 is interesting. Since share is same (ratio 1 : 1), the sales would have been the same if and only if the market was also same (ratio 1 : 1). If the market increased, even marginally, sales of B would have increased and if market decreased, even marginally, sales of B would have decreased.

Market Growth, Individual Growth, Growth in Share

A revision here of concept of multiplying factor relating to a percentage increase/decrease would be appropriate here. So please read the box to recollect the funda learnt in Percentages chapter of Arithmetic book.

Multiplying factor

We have learnt that for every percentage increase/decrease, there exists a multiplying factor, f, such that ...... Initial Value × f = Final Value. Alternately $\frac{\text{Final Value}}{\text{Initital Value}} = f$

For a x% increase/decrease, the multiplying factor is $\left(1 \pm \frac{x}{100}\right)$ E.g. for a 10% increase it is 1.1 or $\frac{11}{10}$; for a 12.5% increase it is $\left(1 + \frac{1}{8}\right) = \frac{9}{8}$; for a 10% decrease it is 0.9 or $\frac{9}{10}$; for a 16.66% decrease it is $\left(1 - \frac{1}{6}\right) = \frac{5}{6}$

Alternately, if the multiplying factor of a percentage change is f, the percentage change can be found as $(f - 1) \times 100\%$. If this is positive, it is a percentage increase and if it is negative it is a percentage decrease.

In the case of the data given in the pie charts, let f_{sales}, f_{share} and f_{mkt} refer to the multiplying factors related to percentage increase/decrease in sales of a company, market share of the company and the total market respectively, we have

We also know that Sales = Market × Share.

Hence $\frac{Sales_{2009}}{Sales_{2008}} = \frac{Market_{2009}}{Market_{2008}} \times \frac{Share_{2009}}{Share_{2008}}$

i.e. $f_{sales} = f_{share} \times f_{mkt}$

This is the relation between the growth rates of the three variables – sales, entire market and market share. Keep in mind, f_{share}, relates to the percentage increase in market share. Thus, if market share increased from 20% to 25%, it relates to percentage increase in the share i.e. $\frac{25\% - 20\%}{20\%} \times 100 = 25\%$.

One could also use the above relation in answering any of questions 1b, 2b or 3b. If sales has to remain constant, then $f_{sales} = 1$.

Thus, in case of A's sales remaining constant from 2008 to 2009, we will have

$\frac{4}{3} \times f_{mkt} = 1 \Rightarrow f_{mkt} = \frac{3}{4}$ i.e. the entire market should decrease by $\frac{1}{4}^{th}$ i.e. 25%. If market decreases by more than 25%, sales of A would reduce (since $f_{sales} < 1$) and if market decreases by less than 25% (or increases) then sales of A would have increased.

This relation between the growth rates in the three variables, $f_{sales} = f_{share} \times f_{mkt}$ is very important. It can also be used to answer questions 4, 5 and 6.

Since f_{mkt} is not known, we cannot find the individual growth rate of sales of any of the companies (ans to Q. 4).

However the sales growth of individual companies, A, B, C, and D would be dependent directly on $\frac{4}{3} \times f_{mkt}$, $\frac{1}{1} \times f_{mkt}$, $\frac{3}{2} \times f_{mkt}$ and $\frac{4}{7} \times f_{mkt}$. Thus, even without knowing the growth rate of the entire market, we can rank the companies in increasing order of the sales growth rate as D, B, A and C (answer to Q 5).

Q. 4a & 4b: If we know that market has increased by 10%, then $f_{mkt} = \frac{11}{10}$ and the individual multiplying factors related to growth of A, B, C and D will be $\frac{4}{3} \times \frac{11}{10}$ i.e. $\frac{22}{15}$, $\frac{11}{10}$, $\frac{3}{2} \times \frac{11}{10}$ i.e. $\frac{33}{20}$ and $\frac{4}{7} \times \frac{11}{10}$ i.e. $\frac{22}{35}$ respectively which translate to a growth of $\frac{7}{15}$, $\frac{1}{10}$, $\frac{13}{20}$ and $-\frac{13}{35}$ i.e. 46.66%, 10%, 65% and –37.14% respectively.

Q. 6: If we know that sales of company C increased by 10%, then for company C, $f_{sales} = \frac{11}{10}$

Using the relation between the three growth rates, we have $\frac{11}{10} = f_{mkt} \times \frac{3}{2} \Rightarrow f_{mkt} = \frac{11}{15}$. Since this factor is less than 1, it implies the market has decreased and the decrease is $\frac{4}{15}^{th}$ i.e. 26.666%

Once we know the growth rate of entire market, we can also find the growth rate of sales of each individual companies, as done in earlier paragraph.

Using $a + b + \frac{ab}{100}$

In percentages we have learnt that whenever we find the net percentage increase by multiplying two multiplying factors, we can also use the formula $a + b + \frac{ab}{100}$. To recap, in any relation of the type C = A × B, if percentage changes in A and B are a% and b%, then the percentage change in the product, C, is $a + b + \frac{ab}{100}$.

A relation of the type C = A ×B is also Sales = Market × Share. Thus, if the percentage change in entire market and the market share of individual company is known, say a% and b%, then percentage change in sales of the company will be $a + b + \frac{ab}{100}$.

Q. 4a: Market has grown by 10% and market share of company A has increased by 33.33%. Thus, the sales of company A has increased by $10 + 33.33 + \frac{10 \times 33.33}{100}$ i.e. 46.66%.

Similarly growth rates for other companies can also be found. But the formula is easy to work only when a% and b% are easy percentages. Alternately, the multiplying factor is a more versatile and easy way even for difficult numbers.

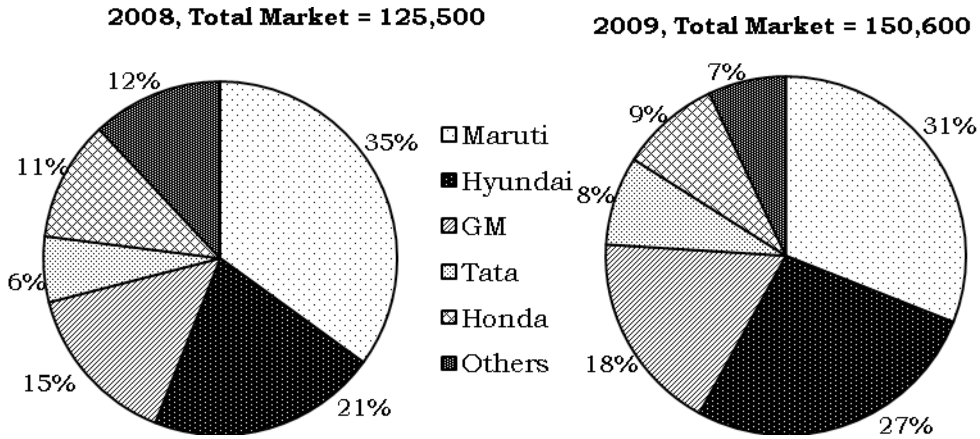
Q. 6: Growth rate of sales of company C is 10% and the growth rate in its market share, from 10% to 15%, is 50%. Assuming growth rate of market as a%, in this question we have $10 = a + 50 + \frac{a \times 50}{100}$ i.e. –40 = 1.5a i.e. a = –26.66%.

Another point worth noting is that in all the above manipulations, we have absolutely no idea about the actual sales values of the market in 2008 and that in 2009 or the actual sales values of any of the individual companies. Yet we can get so many inferences about the growth rates in the sales.

Now for a conventional exercise for you to give an opportunity to use the above learnt manipulations.

Exercise 10:

The pie chart gives distribution of the total sales of cars (in numbers of cars) across the various players who produce cars in India for the years 2008 and 2009. Answer the questions that follow on the basis of only the information given in the graph.



1. How many more cars were sold by GM in 2009 as compared to 2008?

   a. 8,283 b. 8,755 c. 8,056 d. 8,575

2. In 2008, how many more cars were sold by Hyundai as compared to GM?

   a. 7,250 b. 7,380 c. 7,530 d. 7,680

3. What is the ratio of the number of cars sold by Tata in 2008 to the number of cars sold by Honda in 2009?

   a. 2 : 3 b. 5 : 9 c. 5 : 8 d. 4 : 7

4. What is the ratio of the number of cars sold by GM in the year 2008 to those sold in 2009?

   a. 25 : 36 b. 5 : 6 c. 5 : 8 d. 5 : 9

5. From 2008 to 2009, which company grew by the greatest growth rate? (Exclude Others)

   a. Maruti b. Hyundai c. GM d. Tata

6. From 2008 to 2009, which company showed the largest increase in the number of cars sold? (Exclude Others)

   a. Maruti b. Hyundai c. GM d. Tata

7. Which company showed a decrease in the number of cars sold in 2009 as compared to 2008? (Exclude Others)

   a. Maruti b. Honda c. Both Maruti & Honda d. None

8. By what degree is the angle subtended by the sector depicting sales of Tata in 2008 less than the angle subtended by sector depicting sales by Honda in 2009?

   a. 12 degrees b. 10.8 degrees c. 7.2 degrees d. 3 degrees.

9. If Toyota accounts for 25% of Others in 2009, find the market share of Toyota in 2009?

   a. 1.75% b. 2% c. 2.5% d. 3%

10. If Audi accounts for 10% of Others in 2008 and 20% of Others in 2009, find the percentage increase in the number of cars sold by Audi in 2009.

    a. 16.66% b. 20% c. 80% d. 40%

Value, Volume & Price per unit

At the start of this chapter, it was briefly mentioned that the market (and sales of individual companies) can be expressed in two terms - in the example of the car market, we could have said that altogether 20,000 cars were sold. Here we are talking of number of cars and this is called Volume sales. Alternately we could also have said that the total value of all cars sold is Rs. 1,500 lacs. In this case, we are talking of the market in Value terms. Accordingly market share would also specifically mean share of the value of the market or share of the volume of the market.

The linking factor between sales in value terms (in Rs.) and sales in volume terms (in number of products) is the Price per product. Consider that Maruti sold 20,000 cars and this was equivalent to a sales of Rs. 1,500 crores. It would be obvious that ......

Number of cars × Price per car = Value of cars sold.

Thus, in this case, price per car = $\frac{1500 \times 10^7}{20,000}$ = Rs. 750,000

Needless to say, in reality, this will be the average price per car, since there are many different models being sold at different prices.

Now that we know market shares could be in value terms as well as in volume terms, and we also know the relation between Values Sales and Volume sales as price per unit, we can go ahead with a simple exercise that captures all of this.

Exercise 11:

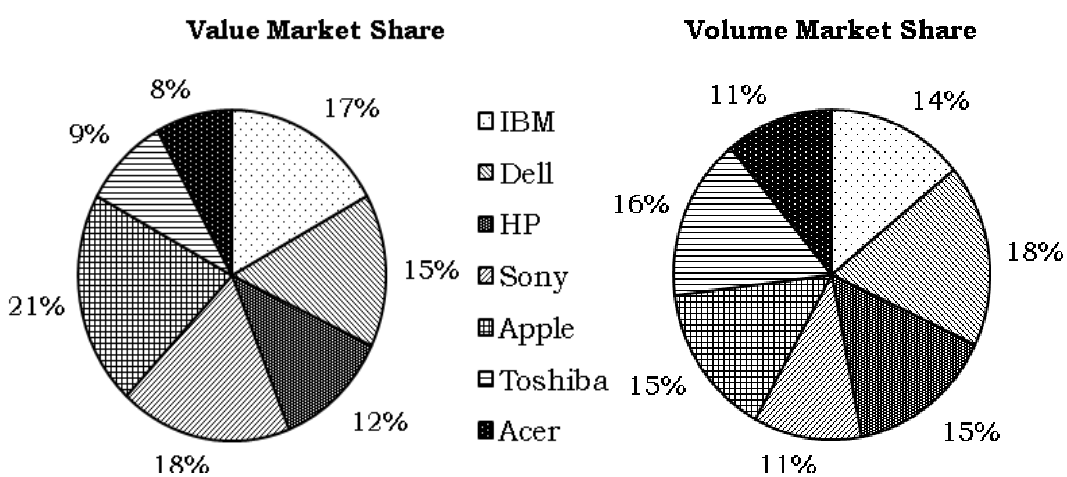
The following table refers to preparatory classes for ACT entrance exam in the city of Poona. The entire preparatory class market in Poona is a fragmented sector with Acumen Classes, Scholar Classes, Topper's Academy, Achiever's Academy and Takshzila being a few players and then there are numerous other players that are all clubbed in as 'Others'. The table gives the data about the number of students at the classes, the revenue (in Rs. millions) and the price per student of their courses. Some cells are intentionally left blank. Your task is to fill in all the blank cells in the table.

	Number of students		Revenue		Product Price
	#	Market Share	Rs. Mn	Market Share	
Acumen Classes					18000
Scholar Classes	2400			30%	
Topper's Academy		9.375%	9		
Achiever's Academy	3200	40.00%			
Takshzila			6	5%	24000
Others	400		3		
Total Market					

Next a more conventional exercise based on the value, volume and price per unit.

Exercise 12:

The laptop market in India is a very competitive market with seven laptop manufacturers being neck to neck in terms of market share. Assume that these are the only sellers of laptops in India. The pie chart to the left depicts the market share of these seven players in terms of the value of the laptops sold whereas the pie chart on the right depicts the market share in terms of the number of laptops sold. Average Market Price of a laptop is the total sales value of all the seven players divided by the total number of laptops sold by all the seven players.



1. If all the laptop sellers are arranged in increasing order of their average price per laptop sold, which model will occupy the middle position?

   a. IBM b. Dell c. HP d. Acer

2. The average price at which Sony sells its laptops is how much percent more/less than the average price at which Apple sells its laptops?

   a. 14.28% b. 15.55% c. 16.45 d. 17.5%

3. Which of the following is larger?

   I. The difference between the average price of a Dell laptop and the average price of a HP laptop

   II. The difference between the average price of a IBM laptop and the average price of a Toshiba laptop

   a. I b. II c. Both are equal d. Cannot be determined

4. As compared to Acer, how many of the other six laptop sellers sell more number of laptops and at an average price that is higher than the average price of Acer?

   a. 6 b. 5 c. 4 d. 3

5. If the Average Market Price of a laptop is Rs. 54,000, find the average price of HP laptop.

   a. Rs. 56,500 b. Rs. 50,400 c. Rs. 48,000 d. Rs. 43,200

6. How many sellers have their average price of a laptop more than the Average Market Price of a laptop?

   a. 2 b. 3 c. 4 d. 5

7. The average price of a Apple laptop is how much percent more/less than the Average Market Price of a laptop?

   a. 40% b. 32.5% c. 14% d. 28.56%

8. The Average Market Price of a laptop is how much percent more/less than the average price of a Toshiba laptop?

   a. 43.75% b. 56.25% c. 77.77% d. 83.33%

Measures using Ratio

Yield, Productivity, Density, Run-rate, Percent of, etc.

Ratios are another very useful measure in Data Interpretation. There are a lot of ratios that are used and you would be familiar with most of them. E.g. in the previous chapter we had ratios of the type $\frac{\text{Number of commercial vehicles sold}}{\text{Total number of vehicles sold}}$ or $\frac{\text{Exports to a country}}{\text{Imports from the country}}$ and many more.

You would all have worked with these ratios without paying any importance to them. These ratios usually boil down to a percentage (or a percent more/less than) and hence these ratios do not pose any difficulty.

Another type of ratio is a measure of efficiency, and this ratio for beginners may seem to be different from the above ratios. Some of the most common such measures are 'yield', 'productivity', 'capacity utilisation', 'density', etc.

A few of these would be exactly similar to the earlier ratios i.e. would be a percentage value e.g. capacity utilisation = $\frac{\text{Total Production}}{\text{Total Capacity}}$. In this case both production and capacity will be measured in the same units and hence the ratio will boil down to a percentage value. (Please take note that in this case, capacity utilisation can also be more than 100% because the entire 'capacity' available may be being worked on for more time (using overtime) or with more speed than prescribed one)

However there are other ratios that may not be a percent value and will also have some unit to them.

E.g. yield of any crop grown is defined as $\frac{\text{Total Production (in kgs, tons)}}{\text{Area under cultivation (in acre, hectare)}}$. Thus, here yield will be measured in tons/hectare or kgs/acre or similar units.

Similarly 'population density' may be defined as $\frac{\text{Total Population (in numbers)}}{\text{Area (in acre, hectare)}}$. Or as used in Science, density can also be $\frac{\text{Weight (in kgs)}}{\text{Volume (in m}^3\text{)}}$.

Any such ratio can be defined

There could be any number of such measures, some may also be very unique or defined by the question setter e.g. 'run-rate', that which is used in Cricket, is also such a measure. Run-rate will be generally defined as $\frac{\text{Runs}}{\text{Overs}}$

For our DI purposes, we need not remember any such measure because all such ratios used will be defined in the data given with the question.

Now, all these ratios have basically just three quantities, viz. the numerator, the denominator and the efficiency measure. So we can have three such situations:

1. Both the numerator and denominator values are given in the table/graph/data. And questions are about the yields

2. The numerator and efficiency is given e.g. the production values and capacity utilisation is given and one needs to find the capacity.

In such cases, it is better to re-arrange the relation such that we get a 'formula' of what we need to find, e.g. $\text{Capacity Utilisation (\%)} = \frac{\text{Production}}{\text{Capacity}} \Rightarrow \text{Capacity} = \frac{\text{Production}}{\text{Utilisation (\%)}}$

3. The denominator and efficiency measure is given e.g. Yield and area under cultivation of crop is given. And questions are on production. Again re-arrange the relation so that one can find the production directly e.g. Production = Yield × Area

These are the only three types of situations. So theoretically there is not much to discuss. However, the difficulty again lies when you have to do the above calculation for four or more times in a single question and then compare the results. So, once again, the way out is to master the calculation techniques learnt in the previous chapter. And remember that you do not need to find the exact values to compare. We will take up two solved sets and then you can practice these types of questions in the 5 sets given for practice.

For the two solved sets, try out the above questions and then read the solutions that follow. See if you have approached the question in the most efficient manner.

Exercise 13:

The following table gives the data about production and area under cultivation for 4 crops across 6 different countries for the year 2006. Production figures are in millions of tonnes and Area under cultivation is expressed in millions of hectares.

Yield for a crop is defined as the ratio of the production value to the area under cultivation for that crop and is expressed in tons/hectare.

Read the table carefully and answer the questions that follow:

	Paddy		Wheat		Maize		Groundnut	
	Production	Area	Production	Area	Production	Area	Production	Area
India	137	43.7	69	26	14.7	7.6	4.9	5.8
China	184	29.4	104	23.4	146	27.1	14.7	4.7
USA	8.8	1.1	57.3	20.3	3.7	1.01	1.5	0.5
Brazil	11.5	2.9	29.5	11.3	42.6	12.6	0.24	0.106
Pakistan	8.1	2.6	21.2	8.4	2.9	1.02	2.2	1.3
World	635	155	606	216	695	144	47.7	22.2

1. In 2006, what was the yield of Maize in Brazil?

   a. 3. 18 b. 3.28 c. 3.38 d. 3.48

2. Which country, among the ones given, had the highest yield of Wheat in the year 2006?

   a. India b. USA c. Brazil d. Pakistan

3. In Brazil, which of the four crops require the least area to produce 1 ton of the crop?

   a. Paddy b. Wheat c. Maize d. Groundnut

4. The yield of Paddy in India was how much percent more or less than the yield of Paddy world-wide?

   a. 32% more b. 32% less c. 24% less d. 24% more

5. If in 2007, the area under cultivation of Groundnut in all the countries remains the same as in 2006, but the yield increases by 10%, what will be the difference in the production of groundnuts in Pakistan and those in USA?

   a. 0.63 b. 0.7 c. 0.77 d. 1.4

6. If in 2007, the production of Wheat world-wide increases by 12.5% and the area under cultivation of Wheat increases only by 10%, what will be the percentage increase in world-wide yield of Wheat?

   a. 2.5% b. 22.375% c. 1.25% d. 2.27%

Solutions:

1. c

Required yield = $\frac{42.6}{12.6}$. One glance at the options will suggest that this will be in the vicinity of 3 and 3.5. A good way to calculate is to express it as mixed fraction and then to approximate the proper fraction involved. Since 126 × 3 = 300 + 78 = 378, we will have $\frac{42.6}{12.6} = 3\frac{48}{126}$. Next, 48/120 is exactly 0.4, thus, the required ratio will be just less than 3.4. This is enough to identify the option choice.

2. b

In this question we have to compare the yields.

One should not immediately start writing all the yields as a fraction. This involves lot of unnecessary writing. One should just glance at the reading and then make some rough approximation. However rough the approximations are, atleast one or two options will be ruled out.

India: Finding orally what multiple of 26 is nearer to 69.

We can orally think that 26 × 2 = 52 and 69 is yet 17 more than 52. Thus, 69 will be more than 2.5 times 26. It will be around 2.68 since 17/26 is just less than 17/25 i.e. 0.68. Now we have a bench-mark, 2.5, to compare other yields with. Remember we want to find the highest yield.

USA: First check to do is to find if the yield is more than the earlier bench-mark, 2.5.

20.3 × 2.5 will be 40.6 + 10.1 i.e. 50.7. But the production of USA is far more than 50.7. In fact the production is 6.6 more, which is considerable. Thus, yield of USA is closer to 3. One can leave it at this level of accuracy or one can also guess that 6.6/20.3 is roughly 1/3 i.e. 0.33 and thus the yield is 2.5 + 0.33 i.e. 2.83.

Brazil: Checking 2.5 times 11.3, we get 22.6 + 5.6 i.e. 28.2. The production is more than this but just slightly. So the yield will not be as much as that of USA.

Pakistan: Checking 2.5 times 8.4, we get 16.8 + 4.2 i.e. 21. The production is almost 21 itself, thus, the yield will be very very marginally more than 2.5, but not as much as that of USA.

Thus, one can identify the answer as USA without the need to write all the four ratios and do detailed conventional calculations.

3. a

The least area to produce 1 ton will be for that crop which has the highest yield. Thus, again yields have to be compared and the highest has to be found.

As learnt in above question, don't write all four yields, just make an estimate first.

Paddy: 11.5 is how many times of 3? Almost 4 times, but lesser than 4.

Wheat: 29.5 is how many times 11.3? Between 2 and 3 times, closer to 3. So this option is ruled out.

Maize: 42.6 is how many times 12.6? This ratio has been found earlier in Q. 1. as 3.8. So this option is also not as great as Paddy and is eliminated.

Groundnut: 0.24 is just 2.4 times 0.1. So this option is also ruled out.

Thus, again without need of detailed calculation, the answer can be found.

4. c

Yield of Paddy in India is 137/43.7

First one should make a rough approximation, by taking round figures if necessary. 137 is more than thrice of 44. Now, one can do more precise calculations with the decimal values. 3 × 43.7 = 131.1. Thus, $\frac{137}{43.7} = 3\frac{5.9}{43.7}$. The fraction part can be approximated in multiple ways, work with whatever comes to your mind first e.g. 6/42 is 1/7 i.e. 0.14. So the fraction part in this case will be lesser than 0.14. Alternately 0.1 times 43.7 is 4.37. Thus the fractional part is little more than 0.1. In either ways the ratio can be found as 3.1 very easily.

World-wide yield is 635/155 i.e. 127/31

Since 31 × 4 = 124, the ratio is $4\frac{3}{31}$ i.e. almost 4.1

We need to find the percent by which 3.1 is more/less than 4.1. And the answer is it is less by 1/4.1 i.e. just less than 25%.

5. c

Since Production = Area × Yield, if the area remains the same and yield increases by 10%, then the production will also increase by 10%.

Rather than find the increased production values of Pakistan and USA, which will need two additions of 10%, and then find the difference, one should realise that the difference will also increase by 10% over the current difference. Thus, the current difference in the production is 0.7 and after a 10% increase it will be 0.77.

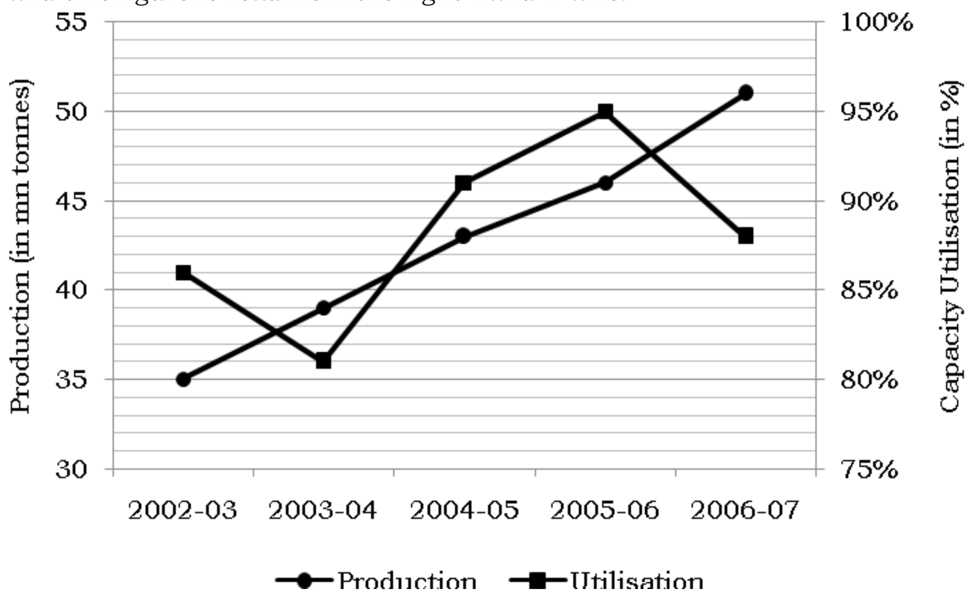
6. d

This is a question based more on funda of percentages rather than any data given in the table. Since Yield = $\frac{\text{Production}}{\text{Area}}$. Thus, the new yield will be

$\frac{9/8 \times \text{Production}}{11/10 \times \text{Area}}$ i.e. $\frac{9}{8} \times \frac{10}{11} \times \text{Old Yield}$ i.e. 45/44 times old yield. Thus, the yield increases by 1/44 i.e. 9.0909/4% i.e. 2.272%.

Exercise 14:

The following line graph depicts the production of steel (in million tonnes) at M/s FISCO Ltd. The same graph also depicts the capacity utilisation (in %)of FISCO's plant capacity and this figure is read from the right-hand Y-axis.



1. In which year was the capacity of the plant of M/s FISCO Ltd.the highest?

   a. 2002-03 b. 2003-04 c. 2004-05 d. 2006-07

2. In which year did the capacity of the plant of M/s FISCO Ltd. increase by the greatest percentage?

   a. 2002-03 b. 2003-04 c. 2004-05 d. 2006-07

3. In the year 2003-04, if the plant had to achieve a 90% capacity utilisation, how much more production of steel was needed?

   a. 38 mn tons b. 40 mn tons c. 44 mn tons d. 48 mn tons.

4. Which of the following statement/s is/are true?

   I. The capacity of the plant of M/s FISCO Ltd. is continuously increasing over the years 2002-03 to 2006-07

   II. The capacity addition in 2003-04 over that in 2002-03 was greater than the capacity addition in 2006-07 over than in 2005-06

   a. Only I b. Only II c. Both d. Neither

5. If in 2002-03, the production increased by 10% over the previous year and the capacity increased by 20% over the previous year, what was the capacity utilisation in the year 2001-02?

   a. 94% b. 96% c. 98% d. Cannot be determined.

Solutions:

1. d

Capacity will be highest when the production is the highest and when the utilisation is the lowest. Since production in 2006-07 is highest of all the years, and utilisation is lesser than that in 2005-06 and 2004-05, these two years are already eliminated.

Even the year 2002-03 is eliminated when compared to 2003-4, when the production was higher than 2002-03 with lesser utilisation. Thus, the comparison is between 2003-04 and 2006-07. This also to a well practiced student should be a clear visual answer that capacity in 2006-07 is higher because the production is very higher than 2003-04(more than 25% higher). But for those who find comfort in numbers, the capacity in the two years 2003-04 and 2006-07 is $\frac{39}{81\%}$ and $\frac{51}{88\%}$. From the first ratio to the second, the numerator has increased by more than 25% whereas the denominator has increased by by less than 10%. Thus, the second ratio is larger.

2. d

Even this question is a straightforward visual one. Just by looking at the graph, one can eliminate two options. And by just jotting numbers, without any calculation, one can find the answer.

The production has been increasing almost by the same amount, in fact in 2006-07it increased by a slightly larger amount (the slight upward tilt of last segment of line). And the utilisation has fallen just twice in 2003-04 and 2006-07. This would have happened only because of a large increase in capacity. Thus, two obvious choices for large capacity additions are these two years.

Now for the percentage of increase. This can be a little tricky in this case because of very close values.

In 2003-04, production increased by 4/35 i.e. 8/70 i.e. 11.428%. Capacity utilisation decreased by 5/86, i.e. just less than 1/17 i.e. 5.88%

In 2006-07, production increased by 5/46 i.e. just less than 1/9 i.e. 11.11%. Capacity utilisation decreased by 7/95 i.e. a little more than 7/98 i.e. 7.14%.

Thus, in 2006-07, the production increased by almost a similar percentage as increase in 2003-04, but the utilisation decreased by a much larger percentage that the decrease in 2003-04. This would result in the percent increase in capacity to be larger in 2006-07.

Why? Because Capacity = $\frac{\text{Production}}{\text{Utilisation}}$. And thus, $\frac{\text{Capacity}_2}{\text{Capacity}_1} = \frac{\text{Production}_2}{\text{Production}_1} \times \frac{\text{Utilisation}_1}{\text{Utilisation}_2}$

where subscript 2 refers to year in which percentage increase is needed and subscript 1 refers to previous year.

Now, for the two cases when subscript 2 refers to 2003-04 and 2006-07, the ratio $\frac{\text{Production}_2}{\text{Production}_1}$ is almost the same (1.114 and 1.11), whereas the ratio $\frac{\text{Utilisation}_1}{\text{Utilisation}_2}$ is more in the case of 2006-07 than 2003-04. (1/0.942 for year 2003-04 and 1/0.928 for year 2006-07).

3. b

A 81% plant utilisation results in a production of 36 mn tons. An additional 9% utilisation would result in additional 4 mn tons i.e. a 90% utilisation would result in 40 mn ton production.

One does not need to find the capacity and then find 90% of it.

4. d

I. From 2003-04 to 2004-05, the capacity utilisation has increased by a faster rate than the production. This means that capacity has decreased in 2004-05. Understand this well, no calculations are involved. Remember, Capacity = $\frac{\text{Production}}{\text{Utilisation}}$. If the Production increases at a lesser rate than the Utilisation, the entire ratio will decrease as compared to precious ratio i.e. new capacity will be lesser than the previous capacity.

II. This has been found in Q. 2. However there it was percentage increase, here it is increase. Since percentage increase in capacity is highest in year 2006-07 and also the capacity in that period is more than the capacity in early years, the absolute increase in capacity in 2006-07 will also be highest.

In 2003-04, ratio of production, with previous year is 39/35 and ratio of utilisation with previous year is 81/86. Thus, ratio of capacity with previous year is $\frac{39}{35} \times \frac{86}{81}$

In 2006-07, ratio of production, with previous year is 39/35 and ratio of utilisation with previous year is 81/86. Thus, ratio of capacity with previous year is $\frac{39}{35} \times \frac{86}{81}$

5. a

Again this is a theoretical question, rather than based on the data.
Denoting utilisation, production and capacity by U, P and C, we have

$$U = \frac{P}{C} \Rightarrow \frac{U_{02-03}}{U_{01-02}} = \frac{P_{02-03}}{P_{01-02}} \times \frac{C_{01-02}}{C_{02-03}} = \frac{11}{10} \times \frac{5}{6} = \frac{11}{12}$$

$$\Rightarrow U_{01-02} = \frac{12}{11} \times U_{02-03} = \frac{12}{11} \times 86 = 86 + 7.8 = 93.8\%$$

Now over for some practice exercises.

Exercise 15:

The following table gives the details of production of 4 crops in India in the years 2006 to 2009. Kharif and Rabi refer to two different seasons of sowing and harvesting in the year. All the 4 crops are produced in both the seasons and the break-up across the two seasons is also given in the table.

Further the table also gives the yield of the crops across the seasons and the years. Yield is defined as the production of the crop per hectare of area under cultivation.

		2006		2007		2008		2009	
	Season	Production	Yield	Production	Yield	Production	Yield	Production	Yield
Rice	Kharif	74.47	1.91	76.7	1.88	80.2	2.12	78.25	2.05
	Rabi	11.25	3.02	12.6	2.98	12	2.88	12.5	3.15
Jowar	Kharif	4.18	1.03	4.6	0.96	4.88	1.11	5.12	0.9
	Rabi	2.96	0.6	3.4	0.65	3.2	0.7	3.8	0.75
Maize	Kharif	11.44	1.74	10.4	1.76	9.1	1.8	12.5	1.85
	Rabi	2.58	3.22	1.9	2.87	3.5	3.08	3.96	3.2
Pulses	Kharif	4.94	0.45	4.7	0.45	5.11	0.55	4.98	0.55
	Rabi	8.41	0.72	7.9	0.74	7.5	0.75	8.2	0.78

Production figures are in million tonnes and the Yield is in tons/hectare.

1. In the year 2007, for how many of the four crops is the yield in Rabi season more than the yield in the kharif season?

   a. 1 b. 2 c. 3 d. 4

2. In which year did the area under cultivation of Rice in the Kharif season increase by the greatest rate?

   a. 2006 b. 2007 c. 2008 d. 2009

3. Arrange the following in increasing order of their magnitudes

   I. the area under cultivation of Jowar in Rabi season in the year 2008

   II. the area under cultivation of Maize in Kharif season in the year 2006

   III. the area under cultivation of Pulses in Rabi season in the year 2009

   a. I, II, III b. I, III, II c. II, III, I d. III, II, I

4. The area under cultivation of which crop was the highest in the Kharif season of 2007?

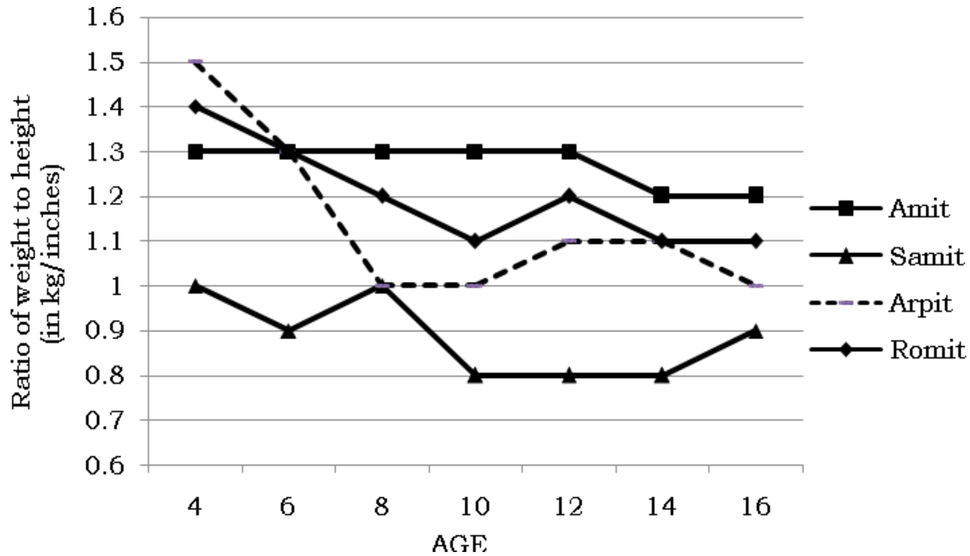
   a. Rice b. Jowar c. Maize d. Pulses

5. The area under cultivation of Jowar in the Kharif season was the least for which year?

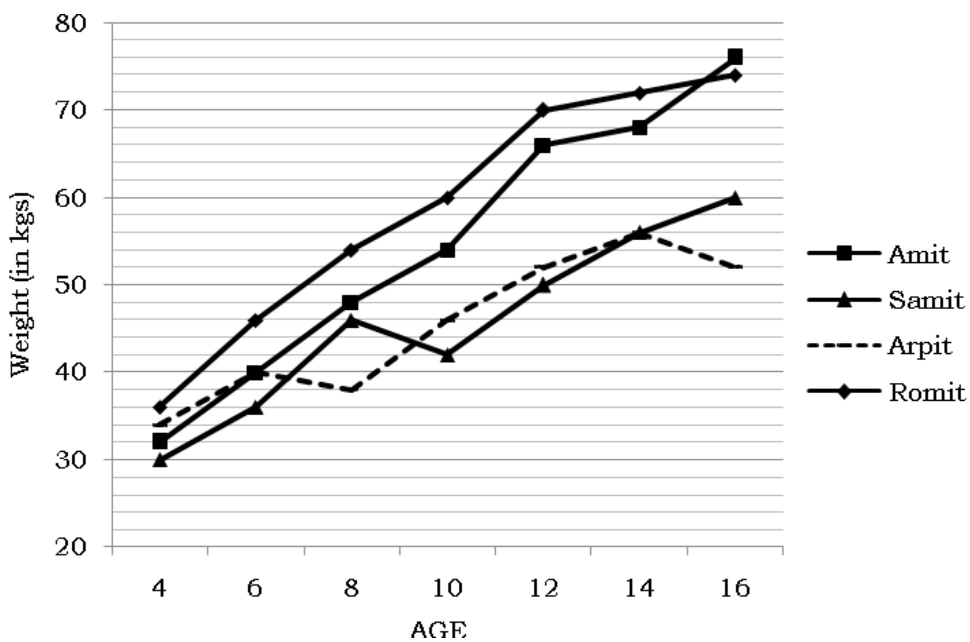
   a. 2006 b. 2007 c. 2008 d. 2009

Exercise 16:

Four children, Amit, Samit, Arpit and Romit participated in a study about the weight and heights of growing children. Their weights and heights were measured every two years between the ages of 4 years and 16 years. The following line graph depicts the ratio of their weights (in kgs) to their heights (in inches).



The following graph depicts the weights (in kgs) with the X-axis being the ages.



1. What was the difference between the heights of Samit and Romit when both of them were 10 years old?

   a. 1 inch b. 1.5 inches c. 2 inches d. 2.5 inches

2. By how many inches did Arpit grow in height from the age of 6 years to 14 years?

   a. 7.5 inches b. 10 inches c. 12.5 inches d. 15 inches.

3. Who among the four children was the shortest at 8 years of age?

   a. Amit b. Samit c. Arpit d. Romit

4. From the age of 4 years to the age of 16 years, who among the four children grew the least in their heights?

   a. Amit b. Samit c. Arpit d. Romit

5. What was the percentage increase in Amit's height between the ages 4 years and 10 years?

   a. 68.5% b. 68.75% c. 69% d. 69.25%

6. How many of the following statement/s is/are true?

   I. The ratio of the height to the weight of all the four children continuously increased or remained the same from 4 years to 16 years of age.

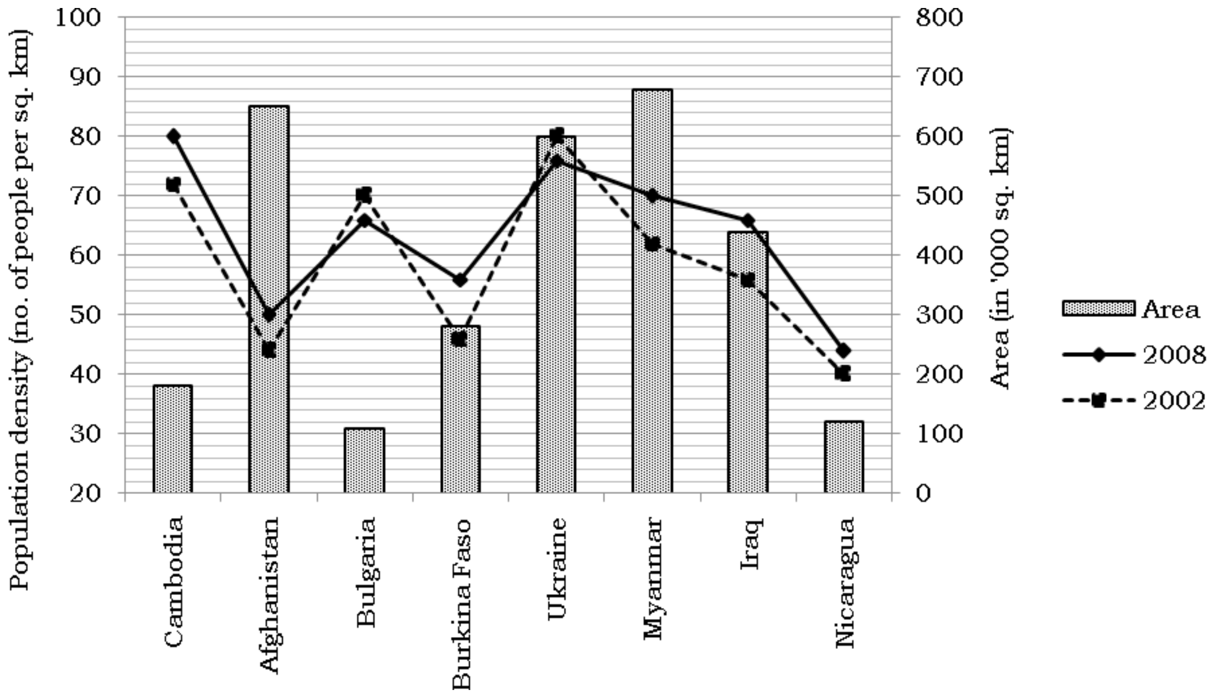
   II. The percentage decrease in the ratio of weight to height of Romit, measured every two years, was constant from 4 years of age to 10 years of age

   III. The height of Arpit has been continuously increasing between the age 4 years to 16 years (measured in every two years interval)

   a. None b. One c. Two d. All three

Exercise 17:

The line and bar graph below depicts the total area and the population density for 8 different countries. The area of the eight countries is the same for the two years 2002 and 2008 and is depicted by the bar graph, read off from the right hand Y-axis. The population density is found by dividing the total population of the country by the area and depicted by the line graphs and is read off the left hand Y-axis. The two line graphs refer to the population density in the year 2002 and 2008.



1. How many countries had a higher population in 2008 as compared to 2002?

   a. 8   b. 7   c. 6   d. Cannot be determined.

2. Find the increase in the population of Myanmar from 2002 to 2008.

   a. 5.44 mn   b. 5.44 lacs   c. 5,440   d. Cannot be determined

3. Find the percentage increase in the population of Burkina Faso from 2002 to 2008?

   a. 17.5%   b. 17.8%   c. 18.1%   d. Cannot be determined

4. The country with the largest population in 2008 among the given countries is:

   a. Afghanistan   b. Ukraine   c. Myanmar   d. Cannot be determined

5. The area per person (area divided by population) is highest for which country, among those given, in the year 2002?

   a. Afghanistan   b. Burkina Faso   c. Nicaragua   d. Cannot be determined

6. If in 2014, the population of Iraq is expected to increase by 10%, what will be the population density of Iraq in 2014, assuming area of Iraq remains the same?

   a. 74.8 b. 72 c. 68 d. Cannot be determined

7. Which country, among those given, had the highest difference in the population between the years 2008 and 2002?

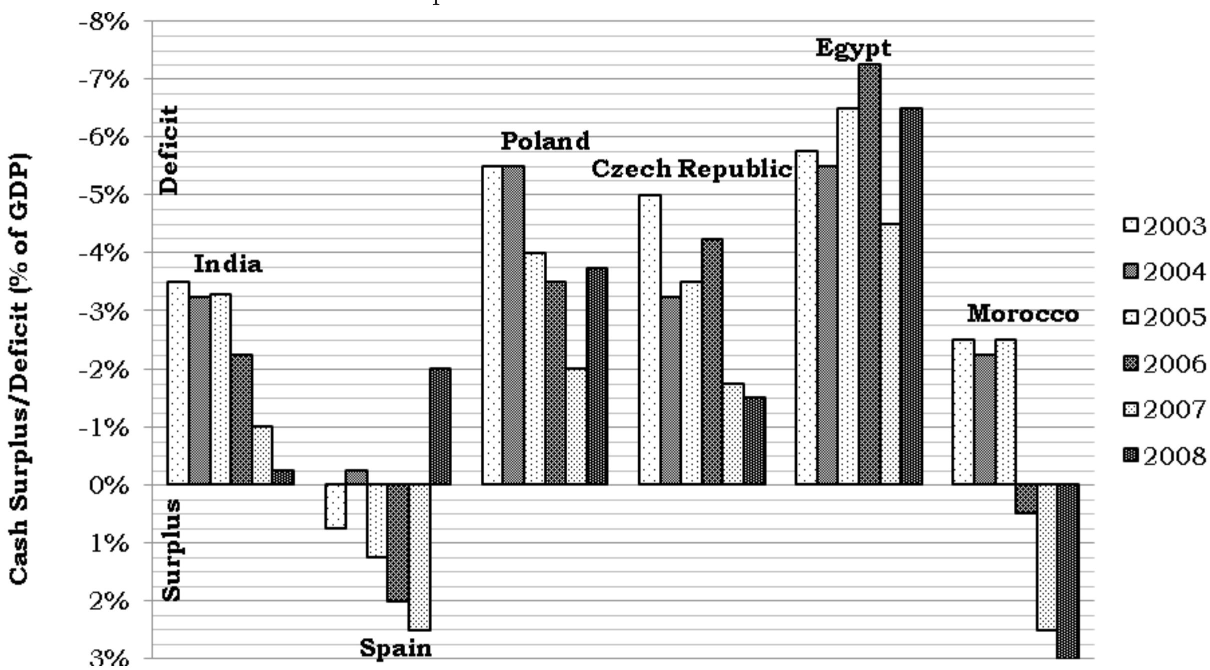
   a. Afghanistan b. Ukraine c. Myanmar d. Cannot be determined

8. Which country's population (among those given) has grown by the largest rate between the years 2002 and 2008?

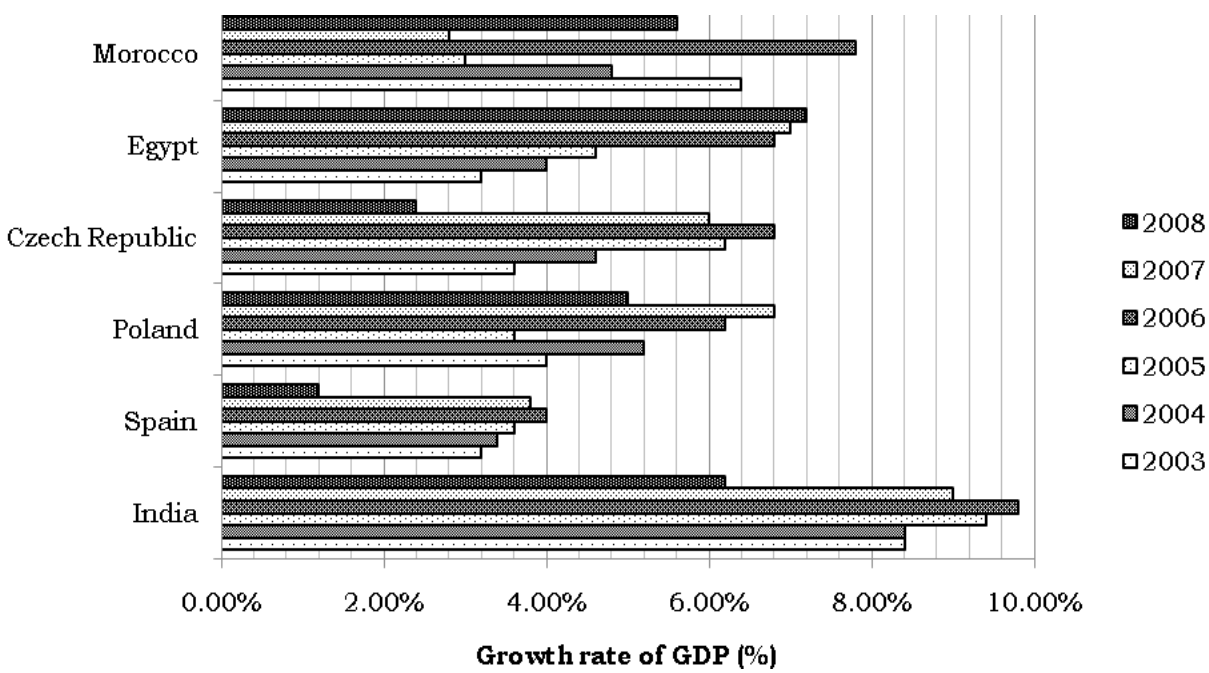
   a. Cambodia b. Burkina Faso c. Myanmar d. Iraq

Exercise 18:

Cash Surplus is the excess of cash revenue over cash expenditure (deficit if this is negative). Rather than the absolute value of this surplus/deficit, it as a percentage of GDP is a more important measure. The following graph gives the cash surplus/deficit as a percentage of GDP for 6 countries for each year from 2003 to 2008. Deficits are shown above the X-axis and Surplus below the X axis.



The following graph gives the growth rate of the GDP's of the same countries for the same time interval.



Each of the question below pertains to only the 6 countries for which data has been given in the above graphs.

1. Which country had the highest cash deficit in the year 2004?

   a. Spain b. Egypt c. Poland d. Cannot be determined

2. Which country's cash deficit increased by the greatest rate from the year 2004 to 2005?

   a. India b. Czech republic c. Egypt d. Cannot be determined

3. Find the ratio of the cash surplus of Spain in year 2005 to it's cash surplus in the year 2007.

   a. 1 : 2 b. 1 : 2.15 c. 1 : 2.3 d. 1 : 2.5

4. Find the ratio of the cash deficit of Poland and Czech Republic in the year 2005.

   a. 3 : 4 b. 2 : 3 c. 1 : 2 d. Cannot be determined

5. If the ratio of India's cash deficit in the year 2007 to Poland's cash deficit in the year 2007 was 7 : 5, find the ratio of the GDP's of the two countries in the year 2006.

   a. 3 : 2 b. 5 : 2 c. 4 : 1 d. 3 : 1

6. What was the percentage increase in Egypt's cash deficit from the year 2007 to 2008?

   a. 44% b. 50% c. 55% d. Cannot be determined

7. Which of the following statement is false?

   I. India's cash deficit has been continuously decreasing over the year 2003 to 2009, except in the year 2005.

   II. Egypt's GDP is continuously growing although the growth rate in it's GDP is continuously decreasing

   a. Only I b. Only II c. Both I and II d. Neither I nor II

8. Czech Republic had the highest deficit in which year in the given time period?

   a. 2003 b. 2005 c. 2006 d. 2007

Exercise 19:

The following table relates to five fictional countries, A, B, C, D and E and the arable land in the country. Two types of data is given for each country for the years 1997, 2002 and 2007. One data is the percentage of total land of the country that is arable. And the second data is the arable land divided by the population of the country (hectare per person). Read the data carefully and answer the questions that follow.

	Arable Land					
	1997		2002		2007	
Country	% of total land area	Hectare per person	% of total land area	Hectare per person	% of total land area	Hectare per person
A	12.30%	0.37	11.30%	0.3	13.10%	0.3
B	19%	0.35	23.10%	0.36	24.40%	0.32
C	17.60%	0.27	19.50%	0.26	20.00%	0.27
D	38.90%	0.52	30.80%	0.43	28.40%	0.4
E	13.70%	0.35	17.00%	0.37	19.00%	0.35

Population density is defined as the ratio of population to the total area of the country.

Assume the total area of land in the country to be constant across the years.

1. What was the percentage increase in the total arable land in country A from 1997 to 2007?

   a. 6% b. 6.5% c. 7.5% d. Cannot be determined

2. What was the population density (persons/hectare) of country B in the year 2002?

   a. 0.3 b. 0.33 c. 0.37 d. Cannot be determined

3. In which of the years was the population density of county C the least?

   a. 1997 b. 2002 c. 2007 d. Cannot be determined

4. In the year 2007, which country had the highest population density?

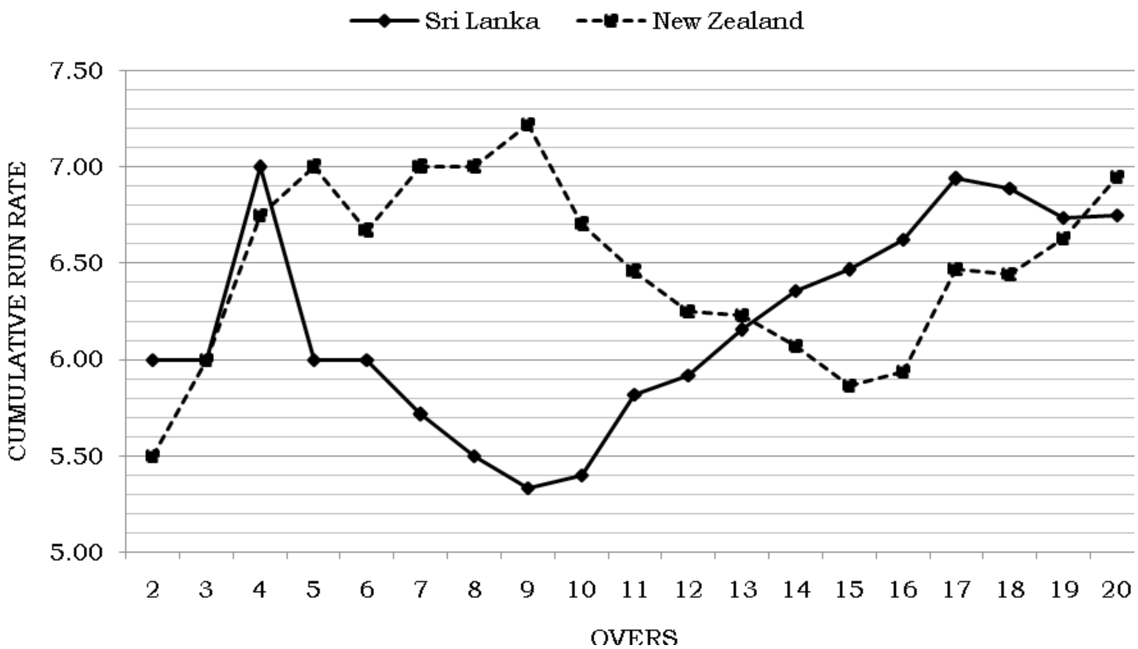
   a. B b. C c. D d. E

5. The population of country E increased by how much percent from 1997 to 2007?

   a. 27.5% b. 32.5% c. 38.5% d. Cannot be determined

Exercise 20:

A 20-20 cricket match was played between Sri Lanka and New Zealand. In the match, each of the two teams bat for 20 overs and score runs. The cumulative run rate of any team is found by dividing the total number of runs scored by the number of overs bowled. The line graph below gives the cumulative run rate of the two teams from the end of 2 overs to the end of 20 overs.



Note: While reading the graph, do remember that the number of runs scored at the end of each over is a natural number. So read the value from the graph intelligently so that the runs scored at end of any over does not turn out to be decimal value.

1. Which of the following question can be answered?
   I. Which team batted in the first inning?
   II. Which team won the match?

   a. Only I b. Only II c. Both d. Neither

2. What was the total score scored by Sri Lanka from the 11th over to the 20th over?
   a. 54 b. 71 c. 81 d. 91

3. In which over did New Zealand score the maximum number of runs?
   a. 17 b. 19 c. 20 d. Cannot be determined

4. The first 50 runs of the Sri Lanka were made in which over?
   a. 7th b. 8th c. 9th d. 10th

5. At the end of how many overs (from the 2nd over to the 20th over) was the score of New Zealand more than the score of Sri Lanka at the end of that over?
   a. 9 b. 10 c. 11 d. 12

6. At the end of which over was the difference between the scores of the two teams, at the end of that over, the maximum?

   a. 8 b. 9 c. 10 d. 16

Index Numbers

Index Numbers are a specific case of a 'measure' using a ratio. Whenever a 'time series' data exists for a very very long time, to make comparisons across the data easier, it is far easier to work on 'indexed' numbers rather than 'actual' numbers. E.g. consider the following data of sales values over 5 years (though sales values and a 5 year period do not really necessitate sales to be converted to index numbers, but the example will give you the basic idea)

Year	2005	2006	2007	2008	2009
Sales	242	290.4	387.2	363	435.6

Comparing the sales data is going to be a bit cumbersome. By comparing, in DI, we lamost always means finding the percentage changes between two time period or finding the ratio of sales in different years.

On the contrary if the sales were expressed as follows, is the comparison between sales of any two years easier?

Year	2005	2006	2007	2008	2009
Sales	100	120	160	150	180

I bet, almost everyone would have found making any sort of comparison far easier with such numbers, right?

And to let the cat out of the bag, this data exactly mirrors the percentage changes in sales as expressed earlier. Check the following, how the 'absolute' sales values have been converted to 'relative' sales values for easier conversion ……

We **proportionally scale** the sales values such that the first reading becomes 100.

Year	2005	2006	2007	2008	2009
Actual Sales Values	242	290.4	387.2	363	435.6
Sales values 'scaled' so that first reading is 100	$242 \times \frac{100}{242}$ = 100	$290.4 \times \frac{100}{242}$ = 120	$387.2 \times \frac{100}{242}$ = 160	$363 \times \frac{100}{242}$ = 150	$435.6 \times \frac{100}{242}$ = 180

Since all the sales values have been multiplied with the same 'scaling factor', the percentage changes (or ratio) between any two reading will be the same as that between the original values.

This new set of data (100, 120, 160, 150, 180), does not refer to the actual sales values. But it mirrors the changes in the sales values exactly. These are hence called Sales Index.

Just knowing the Sales Index, and having absolutely no idea about the 'underlying' sales values, is enough for us to compare the sales in any two years in percentage terms or ratios. This is the use of Index numbers – to make comparisons across years very easy.

Practical Use of Index

There are many Indices in practical use all over the world. Some of the most popular ones are

1. Consumer Price Index (CPI)
2. Index of Industrial Production (IIP)
3. Sensex, the Bombay Stock Exchange's Sensitive Index.

Consider the following data about CPI,

Mar 2009	Jun 2009	Sep 2009	Dec 2009
148	153	163	169

Now, since the data is about prices, does it imply 148 is the price of a particular item? Of what item?

These questions are irrelevant. What the Price Index achieves is to compare the prices.

From March to December, the price levels increased by $\frac{21}{148} \times 100\%$.

Infact if you understood the example of sales completely, you would realise that 148 is not the 'actual' price. The prices are scaled, usually lower down such that the value in some year is 100. This 'some' year is called Base Year. For the CPI data of India, the base year is 2001. Thus, the price levels in 2001 were 'scaled down' to 100. Thus, in Dec 2009, the price levels are 69% more than the price levels in 2001.

Please pay attention to the fact that we have no idea of the actual prices. And we even have absolutely no idea about prices of what and how is the price index calculated. Yet this does not stop us from understanding the percentage changes in prices. This is the perfect utility of an Index.

Index can be constructed for any underlying variables – sales, profit, stock quantity, etc. Further, there are numerous ways in which a Index can be constructed. In the practical situations as outlined in the above box, the Index is constructed in a very elaborate manner. However in situations that we will encounter in the exam scenario, Index is constructed in a very simple manner, usually a proportionate scaling down (as shown in example of sales).

Base year: Any year in the recent past is taken as the base year. And the value of the index for this base year is almost invariably taken as 100 (because it is easier to compare any number with 100, though theoretically you can choose any value you like). And then the actual values are proportionally scaled.

Points to remember:

Knowing just the Index values, one cannot have any idea about the actual value of the underlying variable.

The percentage change in any two underlying values is same as the percentage change in the corresponding index numbers. And the ratio between any underlying values is same as the ratio between the corresponding index numbers.

If we get to know the actual value of the underlying variable for any one year, the value of the variable for all the years can be found out, because we now know the 'scaling factor'

Exercise 21:

The following table gives the index values for the revenues and the number of cars sold by M/s Automatix Ltd. The base year for both revenues and number of cars is 2000. The index values are found using the formula,

$$\text{Index for any year} = \frac{\text{value in that year}}{\text{value in year 2000}} \times 100$$

Year	2000	2001	2002	2003	2004	2005
Revenue Index	100	115	135	160	180	190
Quantity Index	100	112	120	150	180	200

1. Which year was the average price per car the highest?

   a. 2001 b. 2002 c. 2003 d. Cannot be determined

2. In how many years did the revenue increase by a higher rate than the rate by which the number of cars sold increased by?

   a. 5 b. 4 c. 3 d. 2

3. What was the percentage increase in the average price per car from 2000 to 2005?

   a. 0.9% increase b. 5% decrease c. Remains the same d. Cannot be determined.

4. In how many years was the price per car more than the average price per car in the year 2003?

   a. 0 b. 1 c. 2 d. 3

5. If the number of cars sold in 2000 was 1,900 and the revenue earned in the year 2003 was Rs. 120 cr, find the average price per car in the year 2005.

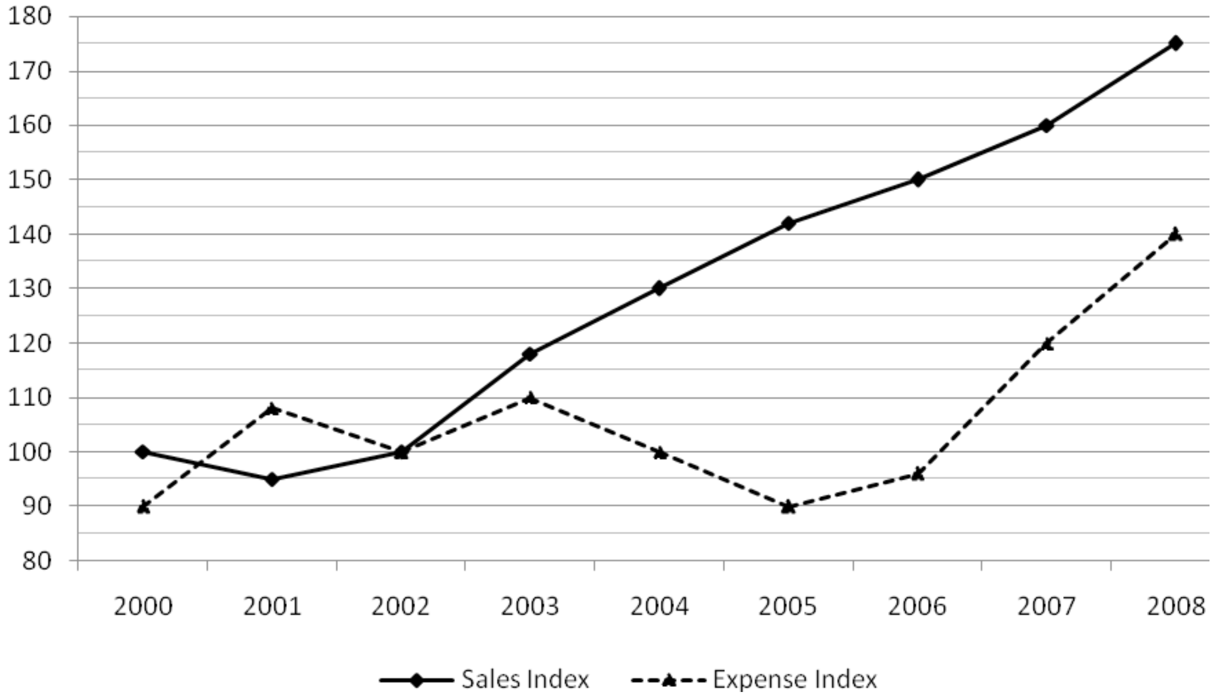
   a. Rs. 3/80 b. Rs. 3.75 cr c. Rs. 3.75 lac d. Cannot be determined

Exercise 22:

The following line graph depicts Index values for Sales and Expenses of a fictional company. The base year to calculate Sales Index is 2002 and to calculate Expense Index is 2004. The index are calculated using the following relations:

$$\text{Sales index in year } X = \frac{\text{sales in year } X}{\text{sales in year 2002}} \times 100$$

$$\text{Expense index in year } X = \frac{\text{expense in year } X}{\text{expense in year 2004}} \times 100$$



Profit of the company = Sales – Expenses and profit percentage is the profit expressed as a percentage of expenses.

1. In how many of the years were Sales more than or equal to the expenses?

   a. 9 b. 8 c. 7 d. Cannot be determined

2. If sales were Rs. 30 lac more than expenses in the year 2004, find the difference between sales and expenses in the year 2005.

   a. Rs. 27 lacs b. Rs. 33 lacs c. Rs. 38 lacs d. Cannot be determined

3. If in the year 2000, the sales and expenses of the company were equal, by what percentage was sales more/less than expenses in the year 2008?

   a. 12.1% b. 12.5% c. 12.8% d. Cannot be determined

4. How many of the following statements are necessarily true?

   I. The profits of the company in 2002 was zero.

   II. The sales of the company was same in the year 2000 and 2002

   III. The percentage by which the sales increased from 2000 to 2008 was more than the percentage by which expenses grew from 2000 to 2008.

   a. None b. One c. Two d. All three

5. If the company had a profit in the year2001, in how many of the remaining years did the company have a profit?

   a. 9 b. 8 c. 7 d. Cannot be determined

6. In how many of the years was the profit amount definitely more than the profit amount of the year 2000? Assume the company made a profit in 2000.

   a. 5 b. 6 c. 7 d. 8

7. In which year did the company make the highest profit? (Assume the company made profit in atleast one of the years)

   a. 2000 b. 2005 c. 2006 d. Cannot be determined

8. In which year was the profit percentage the highest? (Assume the company made profit in atleast one of the years)

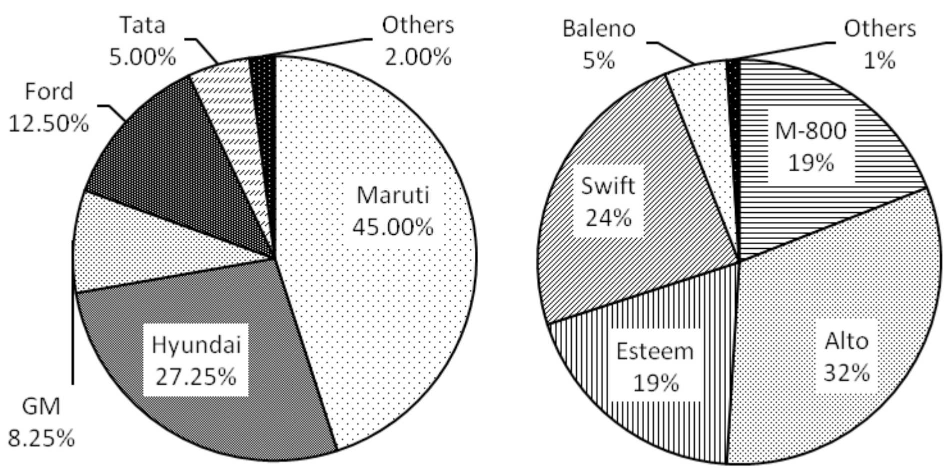
   a. 2005 b. 2006 c. 2007 d. Cannot be determined

Medley of Pie Charts

Pie Charts are very amenable for many varieties of data. However most of them are easy to decipher and now having known some of tougher sets of data, these will seem easy. So we will get on with the exercise directly.

Exercise 23:

The pie chart on the left shows the breakup of the total sales of cars in 2004 across the various car manufacturers present in the market. The pie-chart on the right shows the break-up of the sales of cars of Maruti, a car manufacturer, across the various models that it sells.



Car market in 2004, total sales: 18,000 **Break-up of Maruti across its models**

1. How many M-800 were sold in the year 2004?

   a. 1439 b. 1539 c. 1639 d. 1739

2. If Maruti-Omni accounted for 55.55...% of the 'Others' category of Maruti, find the number of Maruti-Omni sold in 2004.

   a. 45 b. 36 c. 24 d. 18

3. Find the ratio of the number of Alto sold to the number of Ford sold in the year 2004.

   a. 144 : 125 b. 160 : 3 c. 576 : 5 d. 80 : 3

4. If the number of Esteems sold increases by 900, find the market share of Maruti in the car market. (Assume sale of all other models remain the same)

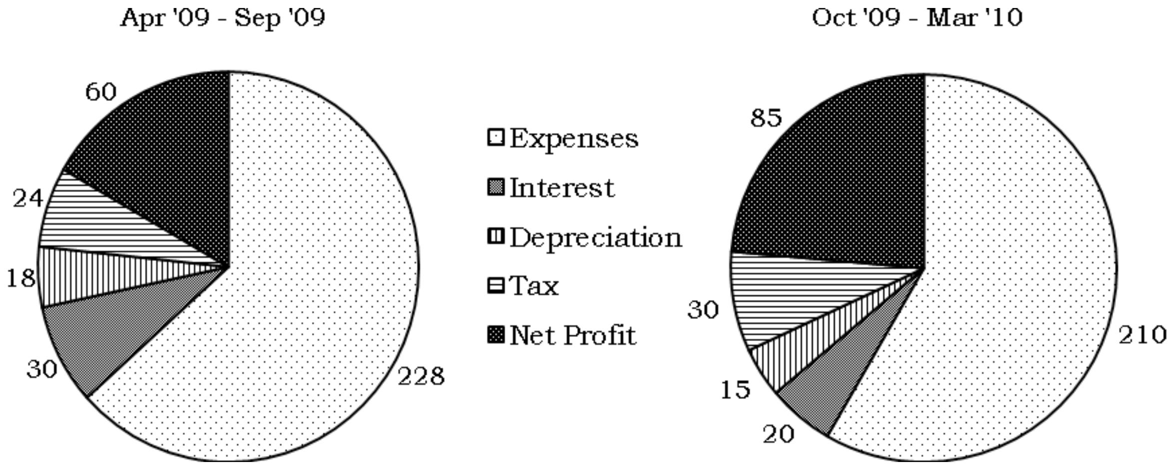
   a. 45.5% b. 46.5% c. 47.6% d. 48.7%

5. If the number of Swifts sold increases, such that it now accounts for 30.909% of the sales of Maruti. Find the number of increase in the Swift sold.

   a. 750 b. 800 c. 810 d. 850

Exercise 24:

The two pie charts show the break-up of Income across various heads for DISKI Ltd in the financial year Apr ’09 to Mar ‘10. To analyse the performance of the company in the first half of the financial year vis-à-vis the second half of the financial year, the pie charts are prepared separately for the two half years. First half of a financial year is Apr to Sep and second half is Oct to Mar.



NOTE: The numbers given besides the sectors are the angle the sector makes at the center of the circle.

1. If the company pays equal amount of tax in the two half years, find the ratio of the income of the company in the first half year to that in the second half year.

   a. 1 : 1 b. 4 : 5 c. 5 : 4 d. Cannot be determined

2. If expenses in the first half year is 60% of the total expenses of the year, which of the following ratios most closely represent the ratio of the Interest paid by the company in the two half years.

   a. 1 : 1 b. 3 : 2 c. 2 : 1 d. 3 : 1

3. If Depreciation accounts for 4.66% of the total Income of the company over the entire financial year Apr ’09 to Mar ’10, Taxes account for what percent of the total Income of the company over the entire financial year

   a. 7% b. 7.33% c. 7.66% d. 8%

Additional directions for questions 4 to 5: Income of the company in the first half year i.e. Apr ’09 – Sep ’09 is Rs. 120 cr and the income in the second half year i.e. Oct ’09 to Mar ’10 is Rs. 144 cr.

4. If profit percentage is Net Profit as a percentage of Expenses, find the profit percentage of for the entire year Apr ‘09 – Mar ‘10.

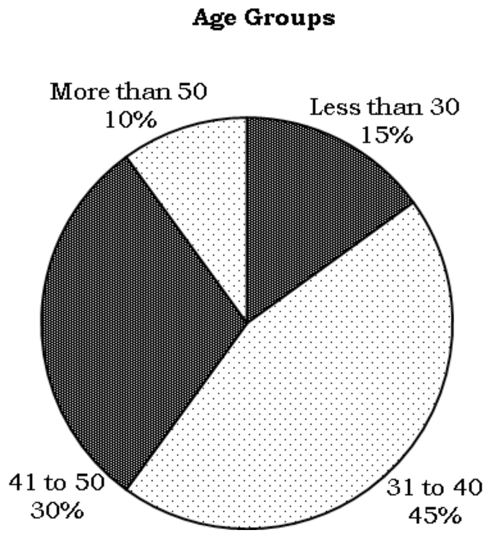
   a. 25% b. 27.5% c. 32.5% d. 33.75%

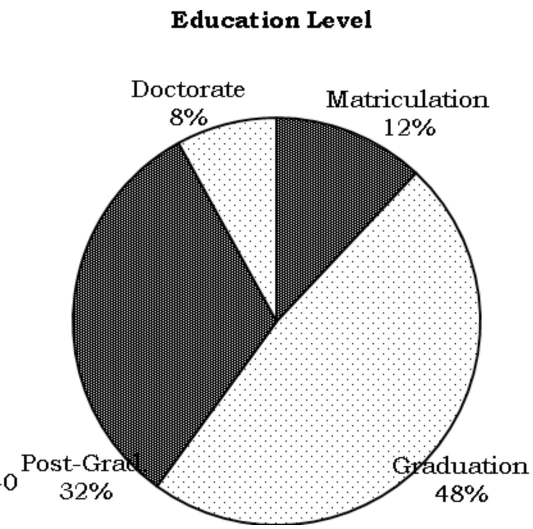
5. By what percentage is the Interest paid in the first half of the year more/less than the Interest paid in the second half of the year?

   a. 25% more b. 20% more c. 20% less d. 25% less

Exercise 25:

The HR department of a company, Source-India, has prepared the following two pie charts related to the employees of Source-India. The pie chart on the left shows the percentage break-up of the employees in different age slabs. And the pie chart on the right does the same but on the basis of the highest education levels of the employee. The hierarchy of education level, from low to high, is Matriculation, Graduation, Post-graduation and Doctorate.





1. If all employees who hold a doctorate degree are above 50 years of age, then what percentage of those above 50 years of age are not doctorates?

   a. 2% b. 80% c. 25% d. 20%

2. If the percentage of employees who are matriculates and less than 30 years of age is 10%, find the percentage of employees who are matriculates or less than 30 years of age.

   a. 7% b. 12% c. 17% d. Cannot be determined

3. Among the employees who are more than 40 years of age, the ratio of the number of employees whose highest education level is Matriculate, Graduate, Post Graduate and Doctorate is 1 : 5 : 3 : 1, find the similar ratio among the employees who are less than 40 years of age.

   a. 2 : 7 : 5 : 1 b. 1 : 8 : 4 : 2 c. 2 : 6 : 3 : 4 d. 2 : 5 : 7 : 1

4. Among the graduates, the ratio of the number of employees in the age groups of less than 30, 31 and 40, 41 to 50 and more than 50 is 1 : 4 : 2 : 1. What percentage of those in the age group 31 to 40 are graduates?

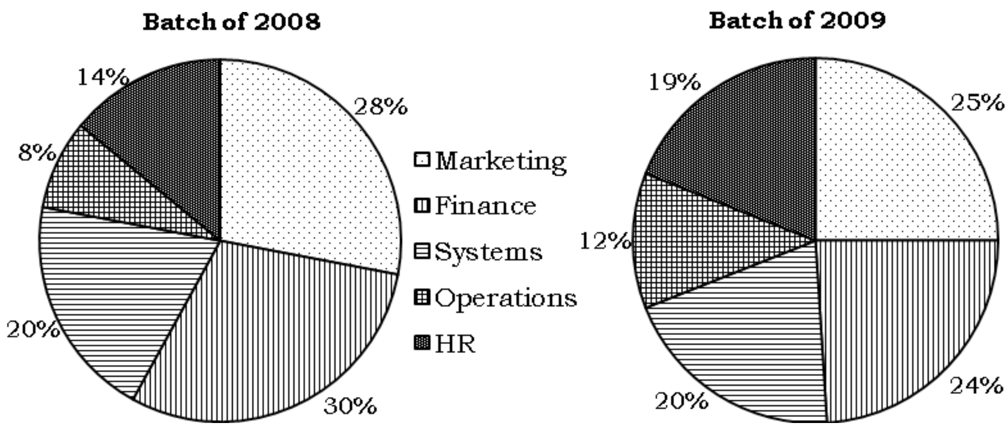
   a. 50% b. 53.33% c. 56.25% d. 60%

5. What minimum percentage of those who are graduates or post graduates are either less than 30 years old or more than 50 years old?

   a. 5% b. 5.625% c. 5.88% d. 6.25%

Exercise 26:

The following is an extract from the placement snapshots at the Top Management Institute of India (TMII). It depicts the percentage break-up of the number of student opting for a job in a particular functional area. The only functional area on offer for placements are Marketing, Finance, Systems, Operations, HR and each student has to opt for only one of these functional area. Also consider that the entire batch was placed, with each student getting exactly one job.



1. If the number of students opting for a job in Finance decreased by 11.11%, by what percentage is the batch strength of 2009 more/less than the strength of 2008?

   a. increased by 11.11% b. increased by 9.99%

   c. increased by 8.88% d. Cannot be determined.

2. Which functional area saw the greatest decline in the number of students opting for it as a career choice? Assume that atleast one functional area showed a decline in the number of students.

   a. Marketing b. Finance c. Systems d. Cannot be determined.

3. If the functional area that saw the greatest percentage increase in number of students opting for it had 20 more students opting for it in 2009 as compared to 2008, find the difference in the batch strength in 2009 and in 2008.

   a. 30 b. 50 c. 60 d. Cannot be determined

4. From 2008 to 2009, if the percentage of people not taking up a job in Systems grew by 10%, by what percent did the number of students taking up a job in Systems grow by?

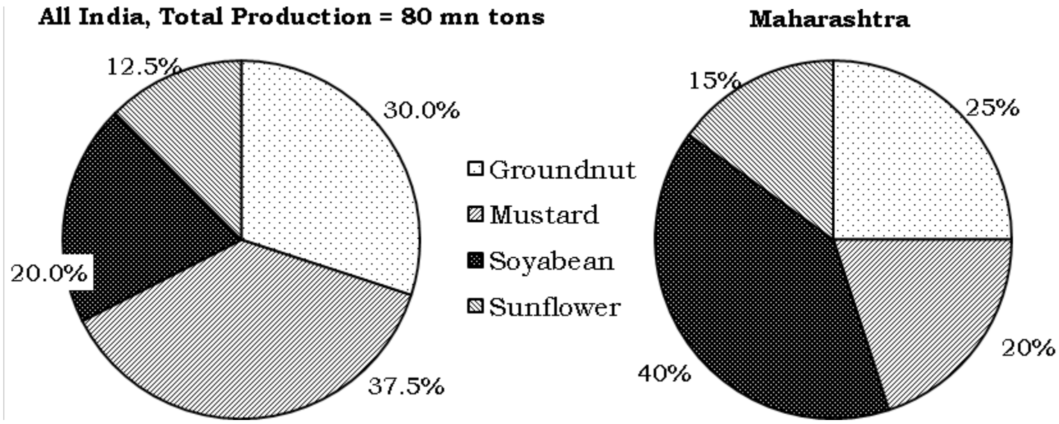
   a. 10% decrease b. no change c. 10% increase d. Cannot be determined

5. If the ratio of the number of students opting for a job in Operations in 2008 to the number of students opting for a job in Finance in 2009 is 2 : 7, what is the ratio of the number of students opting for a job in Marketing in 2008 to the number of students opting for a job in HR in 2009?

   a. 7 : 6 b. 19 : 15 c. 24 : 19 d. 19 : 13

Exercise 27:

The only oil-seeds that are produced in India are groundnut, mustard, soyabean and sunflower. The total production of these oil-seeds in 2008 is 80 mn tons. The pie chart on the left shows the break-up of this production across the four varieties. The pie chart on the right is also the percentage breakup of total production of oil-seeds across the four varieties, but this is only for the state of Maharashtra.



In all the questions “Rest of India” means all states in India other than Maharashtra.

1. If Maharashtra accounts for 10% of the total production of groundnut in India, what percentage of total oil-seed production of India is accounted for by Maharashtra?

   a. 10% b. 12% c. 16.66% d. Cannot be determined

2. If the production of mustard in Maharashtra is 1.5 mn tons, what percentage of total production of oil-seeds in India is accounted for by Maharashtra?

   a. 7.5% b. 8.75% c. 9.375% d. 10%

3. If the ratio of production of Sunflower in Maharashtra and Rest of India is 1 : 4, what percentage of total oil-seed production of Rest of India is accounted for by Sunflower?

   a. 10% b. 12% c. 17.5% d. 20%

4. If Maharashtra produces 40% lesser Soyabean than Rest of India, find the production of Groundnut in Maharashtra.

   a. 2 mn tons b. 3.75 mn tons c. 5 mn tons d. 12 mn tons

5. The maximum production of oil-seeds in Maharashtra can be

   a. 37.5 mn tons b. 40 mn tons c. 50 mn tons d. 66.66 mn tons.

Medley of Bar Graphs

Bar Charts are one of the easiest ones to represent a time-series of any data e.g. sales/GDP/exports/production across years. When sales of only one model across many years is represented, it is a simple bar chart which would not need any explanation. However when there sales of many different models are represented on the same bar chart, some interesting possibilities of representation arises ......

Consider the following data of break-up of exports of services ......

Export of Services across 4 sectors (all values in billion $)

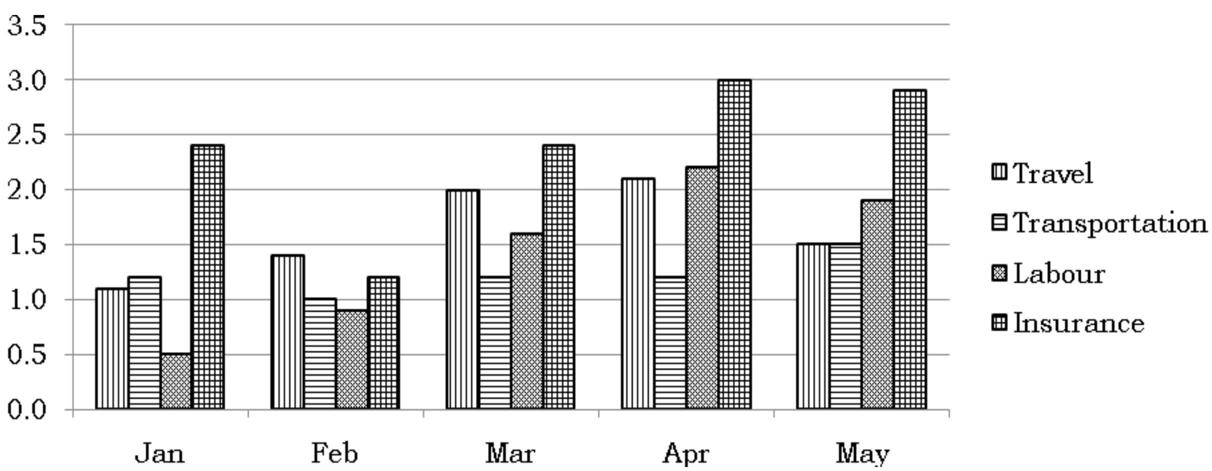
	Travel	Transportation	Labour	Insurance
Jan	0.9	1.1	0.6	2.0
Feb	1.4	0.9	0.9	1.2
Mar	2.2	1.1	1.4	2.4
Apr	2.5	1.1	2.1	2.8
May	2.0	1.6	1.8	2.6

Thus, there is a time series here, Jan to May and the exports of 4 different 'models' – Travel, Transportation, Labour and Insurance are given in the table.

Data similar to the above can also be represented in a bar chart as shown below. The only difficulty now is that the values are not given and have to be read out from the chart.

NOTE: The values used to plot the following graph are different from that of the above table. So while solving the following questions read out the values from the graph itself and not from the above table.

Export of Services across 4 sectors (all values in billion $)



Tip

For any particular question, if you find out the actual reading of any bar, immediately write down the reading on top of the bar.

Further, whenever you have added the total exports of one sector or the total exports of any one month, immediately write the sum so found at an appropriate place (next to the month or next to the sector).

This will save you a lot of time, effort and frustration if you have to use the same reading or the same sum again in another question.

1. How many of the four sectors have shown a decline in exports in any month as compared to the previous month, atleast once in the five given months?

   1. None 2. One 3. Two 4. Three 5. All four

2. In the month of March, which of the four sector has shown the least percentage growth?

   1. Travel 2. Transportation 3. Labour
   4. Insurance 5. Both Transportation & Labour

3. In Apr, exports of labour services account for what percentage of the total exports of all the four sectors?

   1. 22.22% 2. 24.44% 3. 25.88% 4. 27.27% 5. 28.88%

4. Of the total exports of travel services from Jan to May, what percentage was accounted by exports in the month of Feb?

   1. 16.66% 2. 17.3% 3. 17.77% 4. 18.18% 5. 18.54%

Solutions:

For Q. 1: Since we have to look at exports of a sector in successive months, say, travel sector, we will have to look at the first bar of each cluster and see if it has decreased in any month as compared to the previous month.

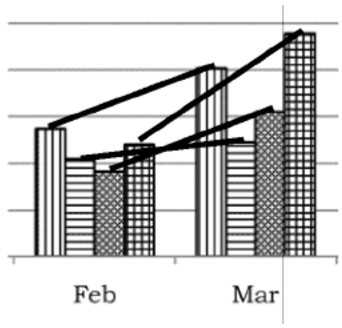
The first bar of each cluster has always grown from Jan to Apr, but in May it is lower than that of Apr. Thus, exports of travel have decreased in May as compared to Apr.

Similarly, comparing the second bar of each cluster, we see that it has reduced from Jan to Feb. Thus, exports of transportation have also decreased atleast once.

Proceeding in the same manner, we can find that both labour and insurance has also shown a decrease in atleast one month as compared to the previous month.

Thus all four sectors satisfy the given condition.

For Q. 2: In this question we have to find the growth between respective bars from the Feb cluster to the Mar cluster.



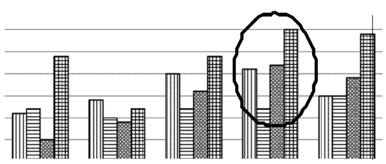
Before calculating the growth rate for each sector, just make a visual check to see if there is any obvious answer. Just imagine a line joining the two points between which we have to find the growth rate. The line joining the bars of transportation sector is the least sloping one by a huge margin and hence obviously has to be the answer.

Thus, required answer is transportation sector.

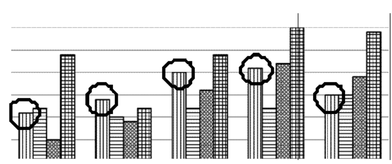
Compare Q. 3 and Q. 4 carefully

In Q. 3 the required percentage is of 'the total exports of all four services in April' whereas in Q. 4 the required percentage is that of 'the total exports of Travel services from Jan to May'.

Thus, for Q. 3, we would have to add all the 4 bars in the cluster of April whereas for Q 4, we would have to add the FIRST bars of all the 5 clusters.



For Q. 3



For Q. 4

For Q. 3: 'Exports of Labour in April' as a percentage of 'total exports of the four sectors in April' $= \dfrac{2.2}{2.1+1.2+2.2+3} = \dfrac{2.2}{8.5}$ i.e. 22/85

10% of 85 = 8.5. What multiple of 8.5 will be just less than 2.2? It has to be the second multiple and will be 17. Thus, 22 has to be considered as 17 + 5.

Next, 5/85 = 1/17 = 5.88%

Thus, 22/85 = 20% + 5.88% = 25.88%

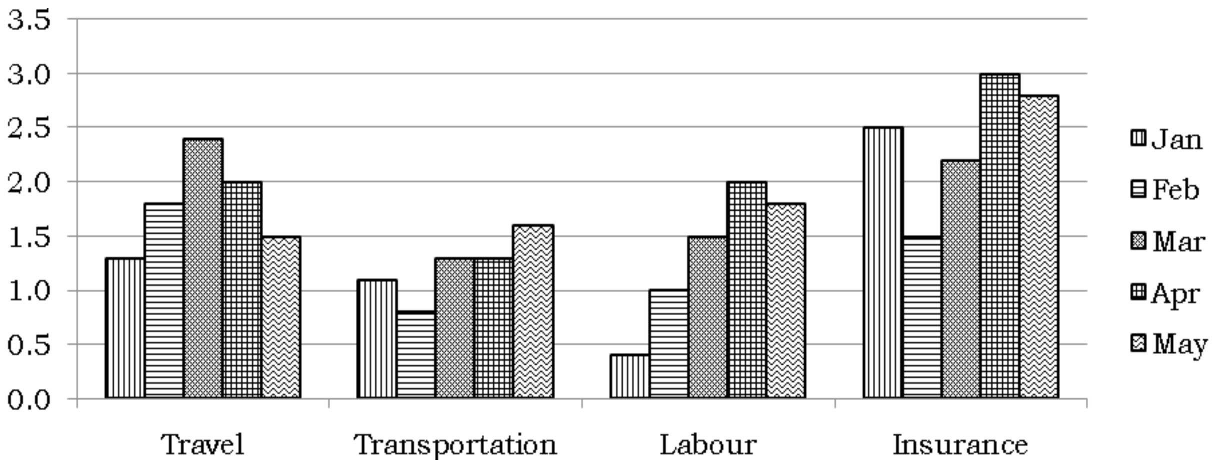
For Q. 4: 'Exports of Travel in Feb' as a percentage of 'total exports of Travel in Jan to May'

$= \dfrac{1.4}{1.1+1.4+2+2.1+1.5} = \dfrac{1.4}{8.1}$.

Approximating it as 14/80 i.e. 17.5%, we can deduce that the correct answer has to be just less than 17.5% and thus the correct answer option is (2)

The following is also a bar chart based on the same data structure (but with differing values so that you do not refer back to the table). Compare it with the above graph to see the difference.

Export of Services across 4 sectors (all values in billion $)



In this graph the clusters are of the sectors whereas the earlier graph had the clusters for a particular month.

Each of the above two type of chart is easy for a different type of birds-eye view. While this type of graph is very easy when we have to compare performance of only one sector across the months, the earlier graph was easy to get an idea about the comparative performance of sectors in a single month.

Compare the following Q. 5, similar to earlier Q. 2. There is hardly any difference between the two questions, it is just the way that data is read from the chart which differs.

5. In the month of April, which of the four sectors has shown the greatest percentage growth?

1. Travel	2. Transportation	3. Labour
4. Insurance	5. Both Labour and Insurance	

6. In which month did exports of labour services show the highest percentage increase over the previous month?

1. Feb	2. Mar	3. Apr
4. May	5. Both Mar and Apr	

7. In which month did the growth rate of total exports of the four sectors show the highest growth rate?

1. Feb	2. Mar	3. Apr
4. May	5. Both Mar and Apr	

8. From Jan to May, in which month was the share of exports of travel services to the total exports of the four services in the month the highest?

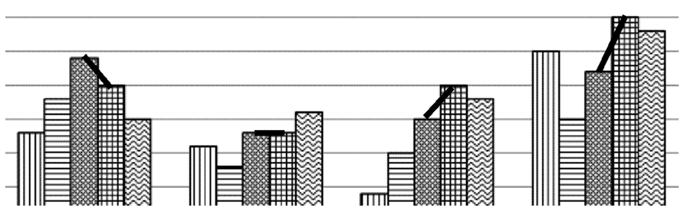
   1. Jan   2. Feb   3. Mar   4. Apr   5. May

9. Compared to the other three sectors, in which month did exports of travel services contribute the largest to the total exports of the month?

   1. Feb   2. Mar   3. Apr
   4. May   5. Both Feb and Mar

Solutions:

For Q. 5: With this type of graph, to find growth rate of sectors in the month of April, we would have to compare the third and the fourth bar for each cluster.



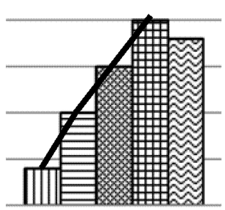
Obviously the only two sectors which can be the answer are labour and insurance sectors.

Labour grew from 1.5 to 2 i.e. a percentage growth of 0.5/1.5 = 1/3 = 33.33%

Insurance grew from 2.2 to 3 i.e. a percentage growth of 0.8/2.2 = 4/11 = 36.36%

Thus, required answer is insurance sector.

For Q. 6: One should avoid the temptation of doing pencil-work as soon as one reads a question. Take a moment to check if the question can be answer through a visual check only. After-all graphs and charts are made just for a quick visual interpretation of the trends.



Labour

It is very evident that the greatest increase in exports has happened from Jan to Feb and also the base is the least. Thus, the highest percentage has to be in the month of Feb.

For Q. 7: For this we would first have to find the total exports of all the four sector in each month. And this can be found by adding the respective bars in the four clusters for each month.

Total exports of the four sectors in Jan = 1.1 + 1.2 + 0.6 + 2.4 = 5.3

Total exports of the four sectors in Feb = 1.4 + 1 + 0.9 + 1.2 = 4.5

Total exports of the four sectors in Mar = 2 + 1.2 + 1.6 + 2.4 = 7.2

Total exports of the four sectors in Apr = 2.1 + 1.2 + 2.2 + 3 = 8.5

Total exports of the four sectors in May = 1.5 + 1.5 + 1.9 + 2.9 = 7.8

Rather than calculate percentage growth in each of the months, a visual check across the values clearly demonstrates that the largest increase was in the month of Mar and it was on a small base of 4.5 in Feb and so Mar has to have the highest percentage growth.

Compare Q. 8 and Q. 9 carefully

In Q. 8, "From Jan to May ...... *share* of exports of travel services *to* the total exports of the month the highest?" implies that we have to find the ratio $\frac{\text{exports of travel service}}{\text{total exports}}$ for all the five months. And we have to find for which month is this ratio the highest.

In Q. 9, "*Compared to the four sectors*, ... exports of travel services *contribute the largest* to *the total exports of the month*" implies that we have find for which month is the ratio $\frac{\text{exports of travel service}}{\text{total exports}}$ more than corresponding ratios for other sectors i.e.

$\frac{\text{exports of transportation services}}{\text{total exports}}$ or $\frac{\text{exports of labour services}}{\text{total exports}}$, etc.

Thus, here we just need to see if the exports of travel services are higher than the exports of each of the other three sectors.

For Q. 8: This question would require the total exports of the four services for each of the five months. An earlier tip already stated that if you have found any total you should write it down very methodically at an appropriate place from where it can be re-covered. We had calculated the total exports of the four sectors for each of the months in the previous question. So using the same values, we have to find which of $\frac{1.1}{5.3}, \frac{1.4}{4.5}, \frac{2}{7.2}, \frac{2.1}{8.5}, \frac{1.5}{7.8}$ is the highest.

From Jan to Feb i.e. from $\frac{1.1}{5.3}$ to $\frac{1.4}{4.5}$, the numerator increases and denominator decreases. Thus $\frac{1.4}{4.5}$ is more than the first ratio.

Since numbers are smaller numbers, one could also think of the remaining four ratios as follows:

$$\frac{1.4}{4.5} \quad \frac{2}{7.2} \quad \frac{2.1}{8.5} \quad \frac{1.5}{7.8}$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$\frac{1}{3.xx} \quad \frac{1}{3.6} \quad \frac{1}{4.xx} \quad \frac{1}{5.xx}$$

Obviously the last two ratios are eliminated. Also a little thinking will tell us that 45/14 is $3\frac{3}{14}$ which is definitely less than 3.6. Thus, the ratio 1.4/4.5 is the highest and required answer is February.

For Q. 9: Looking visually at the first bars of each cluster, we see that in Jan, Insurance was the highest as compared to other sectors.

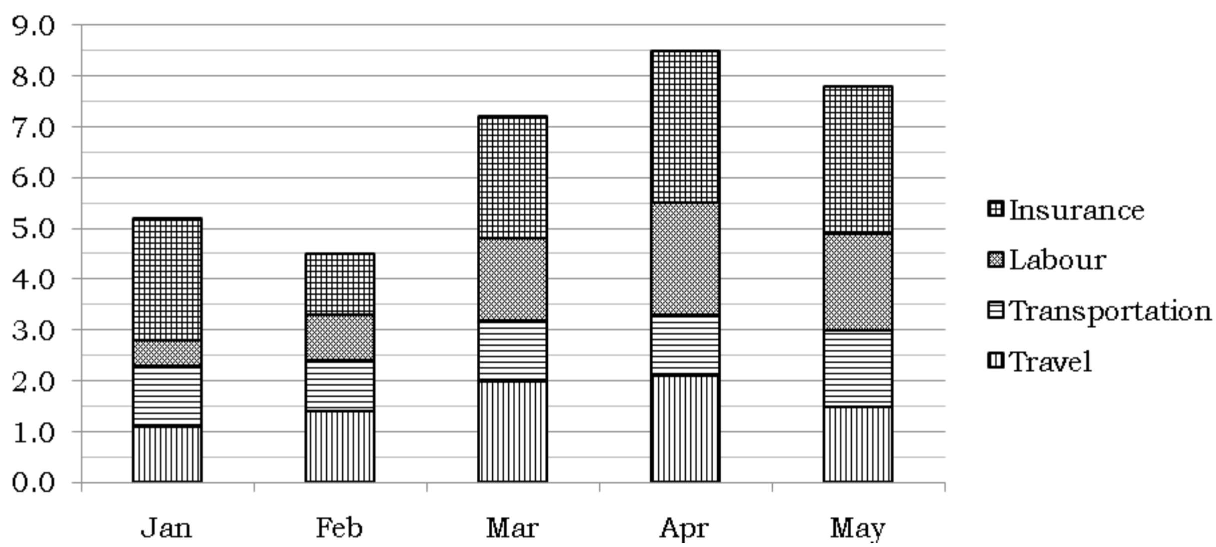
Similarly looking at the second bars of each cluster and finding which sector has the highest of them, next looking at all the third bars of each cluster and finding which sector the highest of them, and so on.

We find that bar corresponding to travel is highest among all the other sectors for both the months of Feb and Mar. Thus, answer is (5).

Cumulative Bar Graph

A similar data could also be represented in the form of a cumulative bar chart as shown below. In this chart, bars of a type (related to a particular month or a particular sector) are stacked one over the other.

Export of Services across 4 sectors (all values in billion $)



With such a data, reading the value for exports for a sector has one more step of subtracting two values. However this representation gives us a better picture of what part of the total exports of the month does each sector account for.

10. By what percentage did export of transportation service grow from Apr to May?

    1. 16.66% 2. 20% 3. 25% 4. 33.33% 5. 50%

11. Which of the four sectors showed the least percentage growth from Jan to May?

    1. Travel 2. Transportation 3. Labour
    4. Insurance 5. Both Transportation and Insurance

12. Which of the following is the ratio of the share of labour service exports in the total exports of the four sectors for the month of Feb to the share of insurance service exports in the total exports of the four sectors for the month of April?

    1. 78 : 145 2. 1 : 2 3. 2 : 3
    4. 17 : 30 5. None of these

Solutions:

For Q. 10: First we will have to find the exports of transportation services in Aril and May.

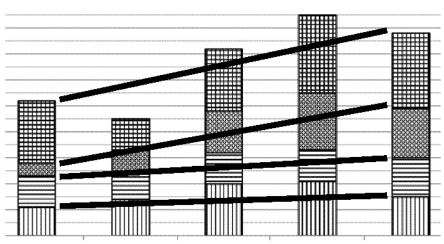
In Apr, exports of transportation services are 3.3 – 2.1 = 1.2.

In May, exports of transportation services are 3 – 1.5 = 1.5

Thus, the required growth rate is 0.3/1.2 = 1/4 = 25%

Before calculating all the values, it would be good to do a visual check to see which options are the likely answers and then do the calculations only for these sectors.

For Q. 11: A visual check will eliminate a few option choices and we would be left with only the likely answers for which calculations would have to be done.



Obviously labour sector grew by the largest magnitude and also having the smallest base would have the highest growth rate. So it is not our answer.

The increments in the exports of travel, transportation and insurance are 0.4, 0.4 and 0.5. And the respective base to calculate the percentage growth are 1.1, 1.1 and 2.4 for the three sectors. Thus, the growth rate is the least for insurance sector.

For Q. 12: In Feb, the share of labour service exports to total exports is $\frac{0.9}{4.5} = \frac{1}{5}$

Similarly, in Apr, the share of insurance service exports to total exports is $\frac{3}{8.5} = \frac{6}{17}$

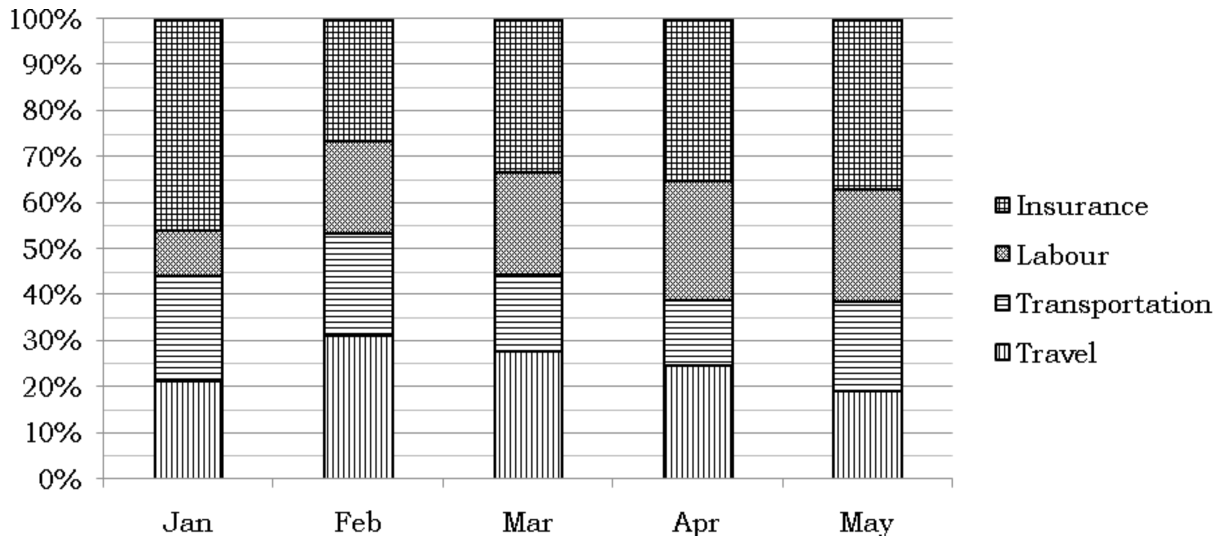
Thus required answer is $\frac{1}{5} : \frac{6}{17}$ i.e. 17 : 30

Cumulative Percent Share Bar Chart

The best way to understand this type of bar graph is by looking at one and then understanding how it is drawn.

The same data structure as all earlier representations had i.e. exports of four sectors from Jan to May (though with different values) is plotted below:

Percentage share of 4 sectors of services in the total export



This type of bar chart has all the bars of the same height i.e. 100% of the underlying value. Thus, in this type of chart, rather than plotting the underlying value of sales / exports / GDP / production / etc, for each model / sector, only the model's/sector's percentage contribution to the total is plotted.

An important thing to note about these graphs then is that we are not aware of actual value of the underlying quantity unless some more info is given.

This is similar to Pie Charts

If this is your reaction after reading the above, you are absolutely right. Just as a pie chart just depicts the percentage share of various products / models / sectors, this bar chart also depicts exactly the same.

Just as the size of the circle in a pie chart has no relevance, the height of the bar in this graph has no relevance.

This type of graph is more space friendlier. We would need to draw five different pie charts for the above data, whereas in the current form it can be presented in a far lesser space.

Thus, as learnt in pie charts, we can compare (find ratio of, percentage difference between) exports of two sectors for any particular month but we cannot compare across months. We do not know what is the total exports across the months. Since we have already learnt pie-charts in details, we are not studying this graph too much in details. Just answer the following questions based on the above bar chart

13. Which of the following statements are necessarily true?

I. Exports of insurance services declined from Jan to Feb

II. If the export of travel services increased from Jan to Feb, then the total exports of the four sectors has grown from Jan to Feb

III. From Jan to May, exports of labour services have shown the highest growth rate (or the least rate of decline) as compared to other sectors.

1. II & III 2. Only II 3. Only III 4. I and II 5. All three

Additional data for Q. 14 & 15: Exports of travel services in January equals exports of insurance services in Apr

14. Exports of transportation services in Apr was what % less than that in Jan?

1. 56.7% 2. 63.4% 3. 67.5%
4. 71.4% 5. Cannot be determined.

15. By what percentage did the total exports of all four sectors change over the period Jan – May?

1. 40% decrease 2. 50% decrease 3. 60% decrease 4. 75% decrease

5. Cannot be determined.

Solutions:

For Q. 13:

Statement I is false because even if the share of insurance service exports has fallen, the actual values could increase if the total exports of the four sector increases by a large value.

Since share of travel service exports has increased from Jan to Feb, actual value of its exports could have increased from Jan to Feb, inspite of the total exports of all the four sectors decreasing (though to a particular limit). And if total exports of all four sectors decreased then the exports of insurance services necessarily decreased (total is decreasing and also its share is decreasing). Thus statement II need not be true.

Statement III is a comparison about percentage growth across the sectors. Whatever happened to the total exports of all the four sectors (increased or decreased) is common to all the four sectors. Thus, that sector which showed the highest percentage increase in its share would also show the highest percentage increase in its exports. Thus, statement III is true.

For Q. 14 & 15:

If total exports of the four sectors in Jan is x and that in Apr is y, then as per data given

21% of x = 35% of y. Thus $\frac{y}{x} = \frac{3}{5}$, implying the total exports of the four sectors in April are 60% of that in Jan i.e. a percentage decrease of 40%. But this is not the answer to Q. 15 as it requires the percentage change from Jan to May and not Jan – Apr. Since we do not have any data about May, the answer to Q. 15 is cannot be determined.

For Q. 14, the ratio of transportation service exports in Apr to that in Jan is approximately

$\frac{14\% \times y}{23\% \times x} = \frac{14 \times 3}{23 \times 5} = \frac{42}{115}$. Thus transportation service exports decreased by 73/115.

10% of 115 = 11.5. What multiple of 11.5 is just lower than 73? The answer is 6^{th} multiple and it is equal to 69. Hence we should look at 73 = 69 + 4.

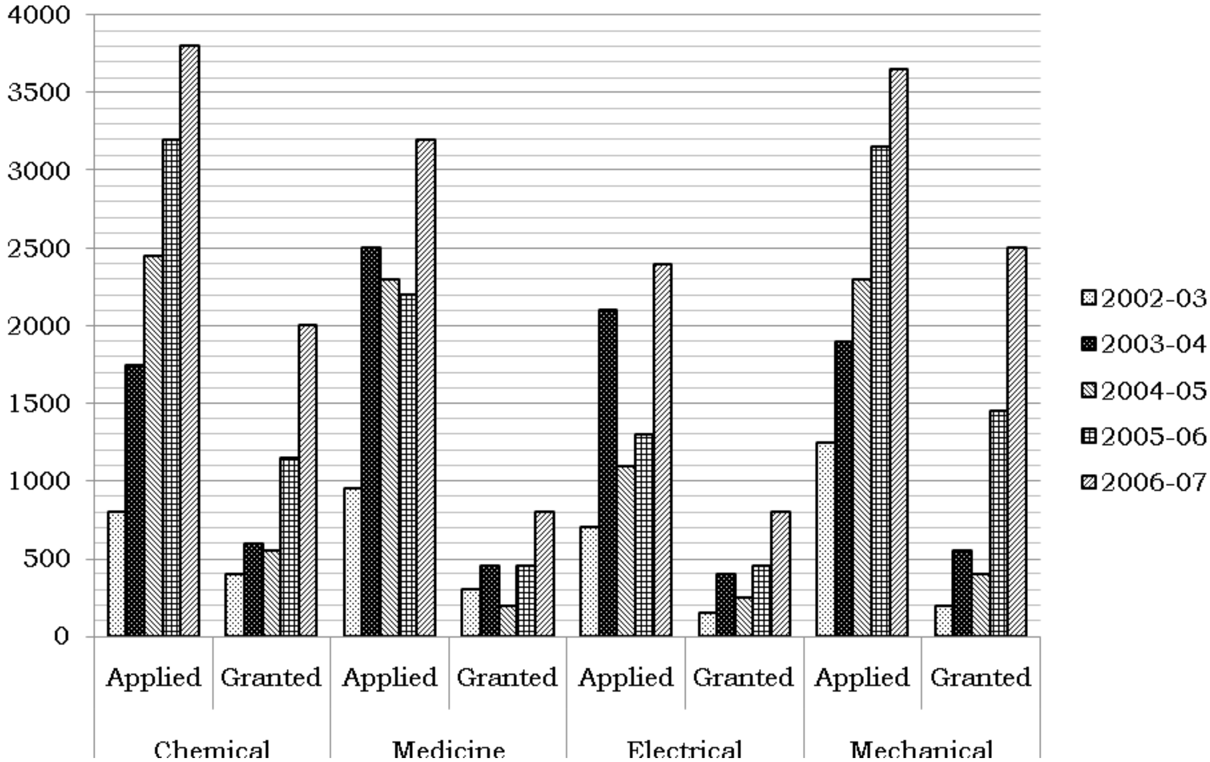
1% of 115 = 1.15. Hence 4 will be a little more than 3%.

Thus, 73/115 will be just more than 63%. And answer will be choice (2)

Now on to some practice exercises for you.

Exercise 28:

The following graph lists the number of patents applied for and the number of patents granted at the US Patent and Copyright Office over the years 2002-03 to 2006-07. Further the patents are always applied in any one of the four categories – Chemical, Medicine, Electrical and Mechanical.



1. In which year was the maximum percentage of patents applied in the field of Medicine granted?

   a. 2002-03 b. 2003-04 c. 2005-06 d. 2006-07

2. In the year 2005-06, for which field was the percentage of applications of patents that were granted, the maximum?

   a. Chemical b. Medicine c. Electrical d. Mechanical

3. In the field of Chemical, in which year did the number of application of patents grow by the largest percentage?

   a. 2003-04 b. 2004-05 c. 2005-06 d. 2006-07

4. For which year and field, was the number of granted patents the least percentage of the number of applied patents?

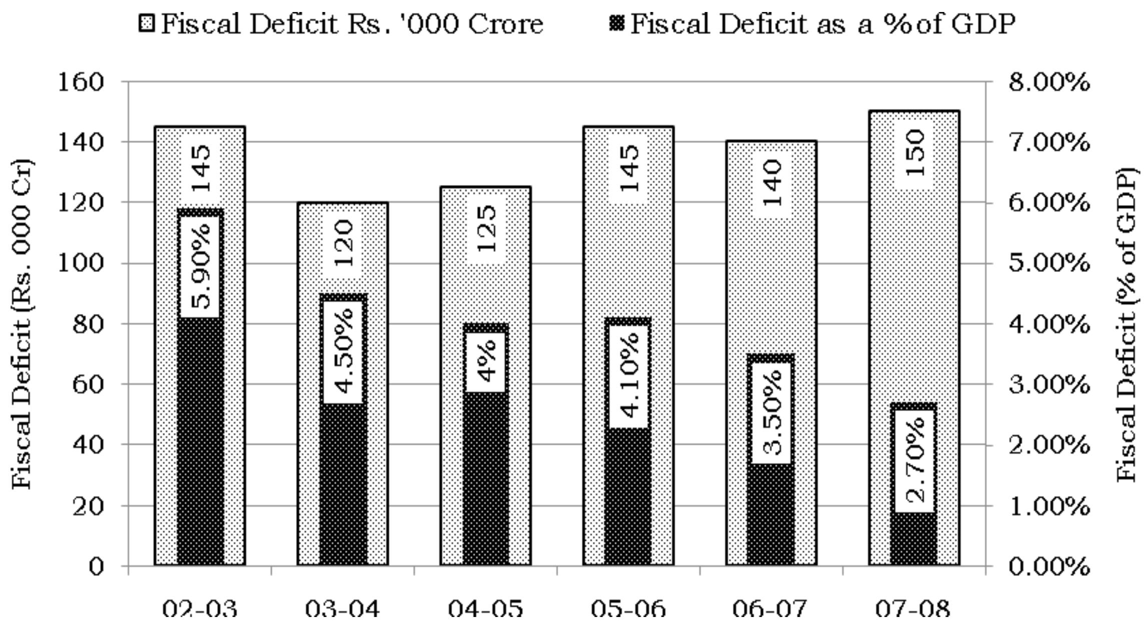
   a. Medicine, 03-04 b. Electrical, 03-04
   c. Mechanical, 02-03 d. Mechanical, 04-05

5. In the year 2003-04, for which field did the number of patents granted grew by the largest percentage, as compared to the previous year?

   a. Chemical b. Medicine c. Electrical d. Mechanical

Exercise 29:

The following graph gives the Fiscal Deficit of India in Rs. '000 crores, which is read from the left hand Y axis. The dark bars also express this fiscal deficit as a percentage of the GDP in that year. The percentage figures have to be read from the right hand Y axis.



1. In which year was the GDP the highest?

   a. 02-03 b. 05-06 c. 06-07 d. 07-08

2. In which year did the GDP grow by the least rate?

   a. 03-04 b. 04-05 c. 05-06 d. 06-07

3. If in the year 08-09, the fiscal deficit increases by 10% and the GDP increases by 5%, then fiscal deficit will be what percentage of the GDP in the year 08-09?

   a. 2.75% b. 2.92% c. 2.83% d. 2.88%

4. In the year 05-06, if the target was to limit the fiscal deficit to just 3.5% of the GDP, then the actual fiscal deficit was how much more than the target (in Rs. 000 cr)?

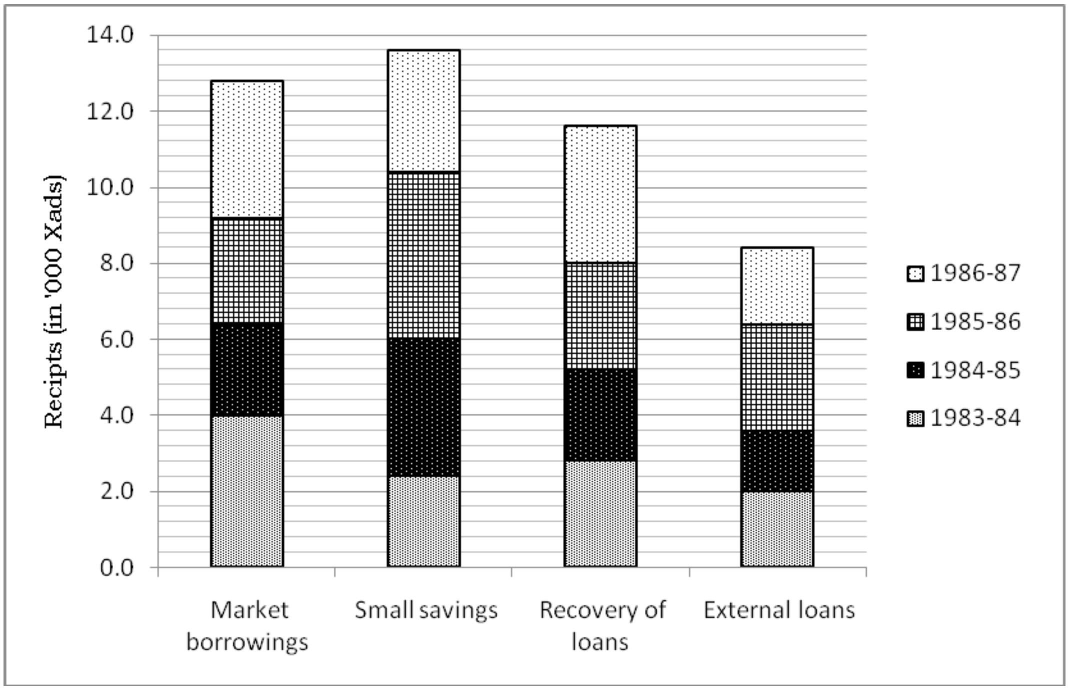
   a. 20 b. 21.2 c. 22.4 d. 24

5. In how many years, as compared to last year, did both the fiscal deficit and the GDP increase, but fiscal deficit as a percentage of GDP decrease?

   a. 0 b. 1 c. 2 d. 3

Exercise 30:

The government of Xanadu classifies it's receipts as Caital Receipts and Revenue Receipts. The following graph gives the figures for all the Capital Receipts that the government receives. The Capital Receipts are categorised as Market Borrowings, Small Savings, Recovery of Loans and External Loans. The receipts for each of these categories, over the years 1983-84 to 1986-87 is given in the following cumulative bar graph. All figures are in thousands of units of the currency of Xanadu i.e. Xad.



1. By what percentage did the capital receipts increase in the year 1985-86?

   a. 26% b. 27% c. 28% d. 29%

2. Small savings accounted for what percentage of the total capital receipts across all the four years considered together?

   a. 31.2% b. 29.3% c. 28.1% d. 27.5%

3. In which year was external loans as a percentage of total capital receipts the highest?

   a. 1983-84 b. 1984-85 c. 1985-86 d. 986-87

4. In the year 1986-87, which of the four avenues of capital receipt accounted for the least percentage of the capital receipts?

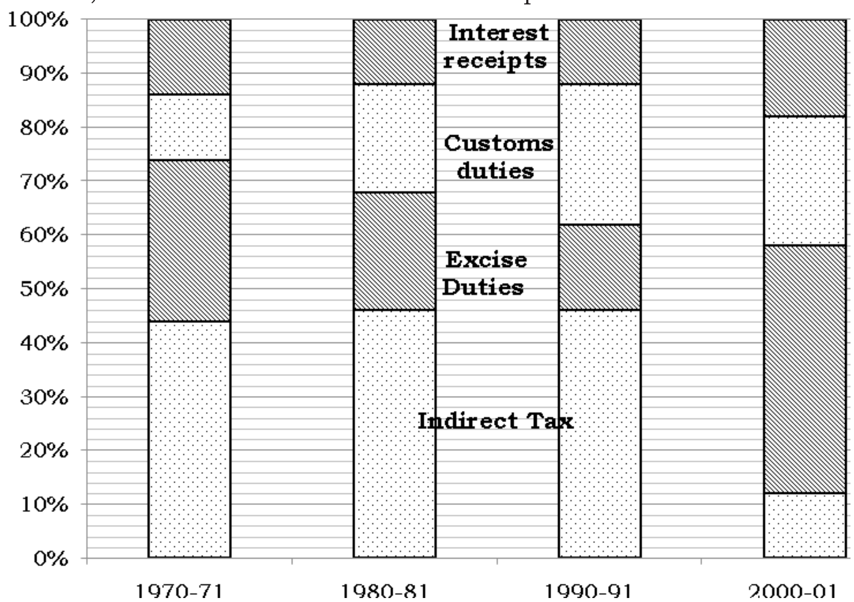
   a. Market Borrowings b. Small Savings
   c. Recovery of loans d. External loans

5. By what percentage did the share of market borrowings in the capital receipts decrease in the year 1984-85 over the previous year?

   a. 40% b. 37.5% c. 32.8% d. 30%

Exercise 31:

The government of Xanadu classifies it's receipts as Caital Receipts and Revenue Receipts. The following graph gives the percentage break-up of the entire Revenue Receipts that the government receives, across the four avenues of revenue receipt i.e. Indirect Tax, Excise Duties, Custom Duties and Interest Receipts.



1. Which of the four avenues of revenue receipts increased by the greatest rate from 1970-71 to 2000-01?

   a. Excise Duties b. Customs c. Interest d. Cannot be determined

2. Which of the following sentences are necessarily true or necessarily false?

   I. Revenues from custom duties in 1980-81 is more than that in 1970-71.

   II. The percentage increase in customs duties from 1970-71 to 2000-01 is more than the percentage increase in excise duties over the same period. (In case both decrease then the percent decrease in custom duties is less than the percent decrease in excise duties)

   a. Only I b. Only II c. Both d. Neither

3. By what percent did interest receipts increase/decrease from 1980-81 to 1990-91 if the percentage decrease in excise duty over the same period was 10%

   a. 34.54% dec. b. 23.75% inc c. 11.65% inc d. Cannot be determined

4. If the income from indirect taxes in 2000-01 was 12.5% more than the income from excise duties in 1990-91, what was the ratio of income from interest receipts in 1990-91 to the income from custom duties in 2000-01?

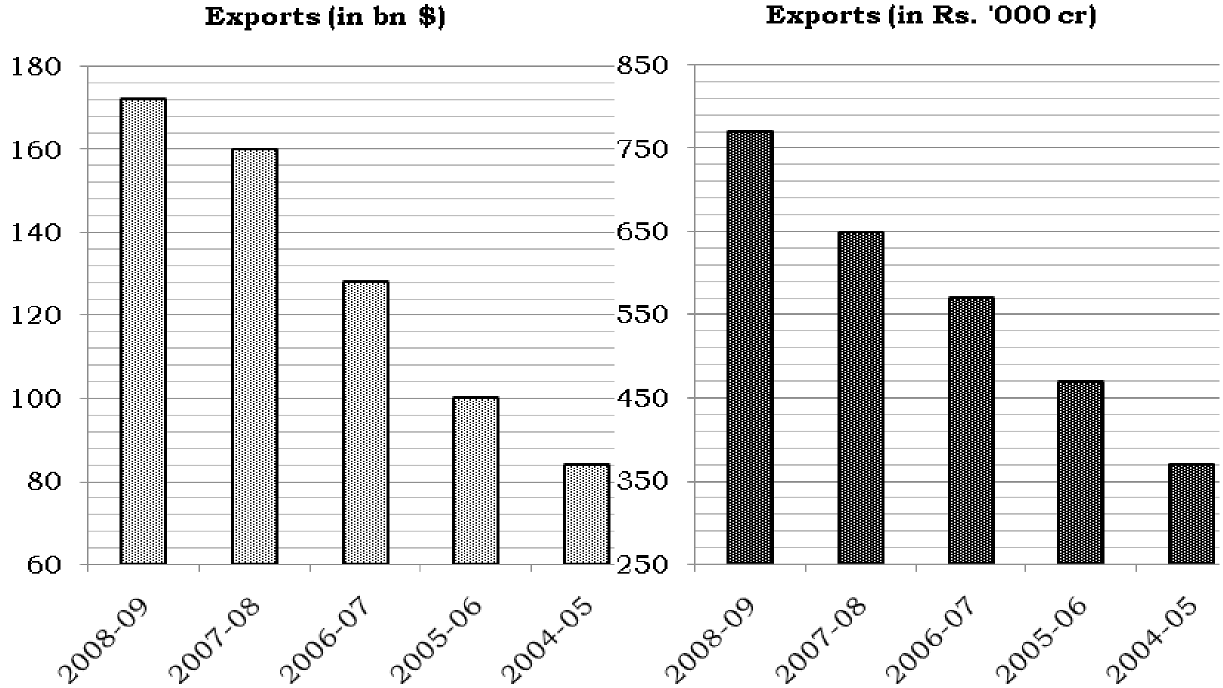
   a. 1 : 2 b. 1 : 3 c. 2 : 3 d. 3 : 4

5. If it is known that revenue from excise duty has continuously increased over the given period, then the least percentage by which income from custom duties would have increased from 1980-81 to 1990-91 would have been:

   a. 30% b. 37.5% c. 67.5% d. 78.75%

Exercise 32:

Both the following graph refer to the exports of India. The bar graph on the left expresses these exports in terms of billions of $ whereas the bar graph on the right expresses these exports in terms of thousands of crore rupees.



1. How many dollars would Rs. 100 fetch in the year 2005-06?

   a. 47 b. 44 c. 2.13 d. 2.5

2. In which year was the value of the dollar (amount of rupees that 1 dollar is worth) the least?

   a. 2008-09 b. 2007-08 c. 2006-07 d. 2004-05

3. In which year did the value of dollar increases by the greatest percentage? Use the meaning of value of dollar as given in the above question.

   a. 2008-09 b. 2007-08 c. 2006-07 d. 2004-05

4. In the year 2009-10, if the exports (in Rs.) increases by 10% and at the same time the value of dollar (amount of rupees that 1 dollar is worth) decreases by 5%, by what percent does exports in (in dollars) increase?

   a. 5% b. 4.5% c. 15% d. 15.78%

5. If in 2004-05, if exports, both in terms of dollar and in terms of rupees, had showed a 10% increase, what was the value of 1 dollar (amount of rupees that 1 dollar is worth) in the year 2003-04.

   a. Rs. 36 b. Rs. 40 c. Rs. 44 d. Cannot be determined.

Vedic Maths

While Vedic Maths is a vast treatise (even covering calculus), we would be learning only three techniques of Vedic Maths viz. to find squares, to find cubes and general rule of multiplication. Because these are the most versatile of all Vedic Math tools and can be applied in any situations. There are many more Vedic Math techniques but then their usage is very limited in scope as they are applicable only in very very limited and narrow conditions. E.g. Base Multiplication – can be applied only when the two numbers to be multiplied as close to each other and also to a base like 10, 100, 1000, etc. or technique for multiplying when one of the multiplicand is a series of 1s i.e. 11, 111, 1111, etc, or a series of 9s i.e. 99, 999, 9999, etc. The above explanation is given so that one does not get over-awed with Vedic Math and only spends as much energy in it as beneficial. Those who are more keen can do well to learn the book Vedic Mathematics by Sri Sankaracharya. There is one more book on faster calculations "The Trachtenberg method of fast calculation". But what follows in this text here is more than sufficient for entrance exam perspective. The above need to be referred only for purely interest purpose.

Finding Squares

Method 1:

For numbers close to 100 i.e. for numbers from 75 to 130:

Here we find the surplus (or deficit) of the number from 100, denoted by x in the following expression and then use the following to find the square.

$(100 + x)^2 = 100 + 2x \mid x^2$

The answer is arrived at in two parts, separated by '|' in the above expression.

The right hand part of the answer has to be exactly two digits. Any extra digit has to be carried forwarded to the left hand part. And if the right hand part is a single digit, it has to be superceded with 0.

E.g.

$103^2 = (100 + 3)^2 = 100 + 2 \times 3 \mid 3^2 = 106 \mid 09 = 10609$

$108^2 = (100 + 8)^2 = 100 + 2 \times 8 \mid 8^2 = 116 \mid 64 = 11664$

$114^2 = (100 + 14)^2 = 128 \mid 196 = 12996$

$119^2 = (100 + 19)^2 = 138 \mid 361 = 14161$

$123^2 = (100 + 23)^2 = 146 \mid 529 = 15129$

$97^2 = (100 + (-3))^2 = 100 + 2 \times (-3) \mid (-3)^2 = 100 - 6 \mid 09 = 9409$

$92^2 = 100 - 2 \times 8 \mid (-8)^2 = 84 \mid 64 = 8464$

$83^2 = 100 - 2 \times 17 \mid (-17)^2 = 66 \mid 289 = 6889$

$78^2 = 100 - 44 \mid 484 = 6084$

For numbers close to 50 i.e. for numbers from 30 to 75:

Here we find the surplus (or deficit) of the number from 50, denoted by *x* in the following expression and then use the following to find the square.

$(50 + x)^2 = 25 + x \mid x^2$

The answer is arrived at in two parts, separated by '|' in the above expression.

The right hand part of the answer has to be exactly two digits. Any extra digit has to be carried forwarded to the left hand part. And if the right hand part is a single digit, it has to be superceded with 0.

E.g.

$51^2 = (50 + 1)^2 = 25 + 1 \mid 1^2 = 26 \mid 01 = 2601$

$59^2 = (50 + 9)^2 = 25 + 9 \mid 9^2 = 3481$

$67^2 = (50 + 17)^2 = 25 + 17 \mid 17^2 = 42 \mid 289 = 4489$

$73^2 = (50 + 23)^2 = 48 \mid 529 = 5329$

$48^2 = (50 + (-2))^2 = 25 + (-2) \mid (-2)^2 = 25 - 2 \mid 4 = 2304$

$39^2 = (50 - 11)^2 = 25 - 11 \mid (-11)^2 = 14 \mid 121 = 1521$

$33^2 = (50 - 17)^2 = 8 \mid 289 = 1089.$

Method 2:

Consider the given number as *ab*, where *b* is the unit digit of number whose square has to be found and *a* is the rest of the number ignoring the unit's digit. E.g. For the number 23, $a = 2$ and $b = 3$, for the number 89, $a = 8$ and $b = 9$ and for the number 117, $a = 11$ and $b = 7$ (in this last example you cannot take $a = 1$ and $b = 17$, *b* has to be the unit digit and *a* the rest of the number). Now the answer is found in three parts as follows...

$ab^2 = a^2 \mid 2 \times a \times b \mid b^2$

Each of the two parts to the right has to have exactly one digit. Any other digits have to be carried forward to the immediate left part.

E.g.

$53^2 = 5^2 \mid 2 \times 5 \times 3 \mid 3^2 = 25 \mid 30 \mid 9 = 2809$

$87^2 = 64 \mid 112 \mid 49 = 64 \mid 116 \mid 9 = 7569$

$112^2 = 11^2 \mid 2 \times 11 \times 2 \mid 2^2 = 121 \mid 44 \mid 4 = 12544$

$176^2 = 289 \mid 204 \mid 36 = 30976$

Squares of numbers ending with 5:

If we want to find the square of a number ending with 5, consider the number as $n5$, where n is the number ignoring the unit digit of the given number. E.g. for the number 45, $n = 4$, for the number 125, $n = 12$ and for the number 215, $n = 21$.

Now you can find the square of the number as follows in two parts...

$n5^2 = n \times (n + 1) \mid 25$

The right hand part is always 25.

E.g.

$35^2 = 3 \times 4 \mid 25 = 1225$

$85^2 = 8 \times 9 \mid 25 = 7225$

$135^2 = 13 \times 14 \mid 25 = 18225$

Finding cubes

Just as we found squares, using the approach of considering the number as ab, we can find cubes as follows... a and b refer to the same as explained in the note on finding squares.

$ab^3 = a^3 \mid 3a^2b \mid 3ab^2 \mid b^3$

Each of the three parts on the right has exactly one digit. Any other digits have to be carried forward to the immediate left part.

E.g.

$16^3 = 1^3 \mid 3 \times 1^2 \times 6 \mid 3 \times 1 \times 6^2 \mid 6^3 = 1 \mid 18 \mid 108 \mid 216 = 4096$

$233 = 2^3 \mid 3 \times 2^2 \times 3 \mid 3 \times 2 \times 3^2 \mid 3^3 = 8 \mid 36 \mid 54 \mid 27 = 12167$

General Rule of Multiplication

This is a columnar multiplication method as explained in the following figure...

In each step, you multiply the two numbers at the end of each arrow and then add all the products so obtained.

The summation would give one digit of the answer at a time, the rest of the digits have to be carried forward.

Consider multiplying 74 × 36

Step 1
$$\begin{array}{lrr} & 7 & 4 \\ & & \updownarrow \\ \times & 3 & 6 \\ \hline \textit{Answer} & & 4 \\ \textit{Carry} & 2 & \end{array}$$

Step 2
$$\begin{array}{lrrr} & & 7 & 4 \\ \times & & 3 & 6 \\ \hline \textit{Answer}: & & 6 & 4 \\ \textit{Carry} & 5 & 2 & \end{array}$$

Step 3
$$\begin{array}{lrrrr} & & & 7 & 4 \\ & & & \updownarrow & \\ & & \times & 3 & 6 \\ \hline \textit{Answer}: & 2 & 6 & 6 & 4 \\ \textit{Carry} & & 5 & 2 & \end{array}$$

Step 1: 4 × 6 = 24 i.e. right most digit of answer = 4 and 2 is carry forward

Step 2: (7 × 6) + (3 × 4) i.e. 42 + 12 i.e. 54 + 2 (carry) = 56 i.e. digit in answer = 6 and 5 is carried forward.

Step 3: 7 × 3 = 21 + 5 (carry) = 26

Thus answer = 2664.

In the case of multiplication of two digit numbers, one can also think of the above as follows:

		Answer	
Step 1:	$7\ 4 \times 3\ 6 =$	4	$4 \times 6 = 24$
carry		2	
Step 2:	$7\ 4 \times 3\ 6 =$	64	$(7 \times 6) + (4 \times 3) = 54 + 2 = 56$
carry		52	
Step 3:	$7\ 4 \times 3\ 6 =$	2664	$7 \times 3 = 21 + 5 = 26$
carry		52	

Now consider, the multiplication of two three digit numbers...say 682 × 195

Step 1

$$\begin{array}{lrrrrrr}
 & & & & 6 & 8 & 2 \\
 & & & \times & 1 & 9 & 5 \\
\hline
Ans: & & & & & & 0 \\
Carry: & & & & & 1 &
\end{array}$$

Step 2

$$\begin{array}{lrrrrrr}
 & & & & 6 & 8 & 2 \\
 & & & \times & 1 & 9 & 5 \\
\hline
Ans: & & & & & 9 & 0 \\
Carry: & & & & 5 & 1 &
\end{array}$$

Step 3

$$\begin{array}{lrrrrrr}
 & & & & 6 & 8 & 2 \\
 & & & \times & 1 & 9 & 5 \\
\hline
Ans: & & & & 9 & 9 & 0 \\
Carry: & & & 10 & 5 & 1 &
\end{array}$$

Step 4

$$\begin{array}{lrrrrrr}
 & & & & 6 & 8 & 2 \\
 & & & \times & 1 & 9 & 5 \\
\hline
Ans: & & & 2 & 9 & 9 & 0 \\
Carry: & & 7 & 10 & 5 & 1 &
\end{array}$$

Step 5

$$\begin{array}{lrrrrrr}
 & & & & 6 & 8 & 2 \\
 & & & \times & 1 & 9 & 5 \\
\hline
Ans: & 1 & 3 & 2 & 9 & 9 & 0 \\
Carry: & & 7 & 10 & 5 & 1 &
\end{array}$$

The same method can be used for a multiplication of two digit and three digit numbers. All one has to do is to consider the two digit number as a three digit number with the leading digit as 0.

It will pay off very well if you start multiplying numbers this way. While the above may seem to be as lengthy as the regular method of multiplication, it is not so. The reason is that you are doing the same multiplication as done in the regular method. The only difference is that you are adding them in your mind and writing the answer in one line So it is strongly advised that you try multiplying numbers using this method and after practicing it for a couple of weeks, then decide for yourself which method is better off.

Now one practice excercise on next page for you to try. Please memorise the approaches and try not to refer back to these pages for the approaches. Answers are given at the bottom of the page.

Exercise:

1. Find the squares of the following numbers.

a. $37^2 = ?$	b. $45^2 = ?$	c. $63^2 = ?$	d. $72^2 = ?$
e. $88^2 = ?$	f. $102^2 = ?$	g. $128^2 = ?$	h. $145^2 = ?$

2. Find the cubes of the following numbers.

a. $12^3 = ?$	b. $15^3 = ?$	c. $31^3 = ?$	d. $103^3 = ?$

3. Find the following products.

a. 48×39	b. 92×37	c. 34×17	d. 83×87
e. 63% of 75	f. 37% of 12	g. 137×953	h. 45×347

Answers:

1.	a. 1369	b. 2025	c. 3969	d. 5184
	e. 7744	f. 10404	g. 16384	h. 21025
2.	a. 1728	b. 3375	c. 29791	d. 1092727
3.	a. 1872	b. 3404	c. 578	d. 7221
	e. 47.25	f. 4.44	g. 130561	h. 15615

Explanations

Exercise 1:

1. b

From 2003 to 2006, sales of C grew from 680 to 880 i.e. an increase of 200 on a base of 680. Thus, total growth rate is $\frac{200}{680}$.

Next 10% of 680 is 68 and hence 200 will be just just less than 3 × 10% i.e. 30%.

But we are asked the simple 'annual' growth rate. From 2003 to 2006, there are 3 annual periods of increment 2003–2004, 2004-2005 and 2005-2006. Thus, the required annual growth rate is 10%.

2. d

Cement market in 2003 = 260 + 1050 + 680 = 1990

Cement market in 2006 = 375 + 1465 + 880 = 2720

Thus, cement market grew by 730 on a base of 1990. And the growth rate is $\frac{730}{1990}$. Assuming the value 1990 as 2000, the above expression becomes 36.5%. Thus, the required answer has to be marginally greater than 36.5% i.e. option choice (d)

3. b

Combined sales of A and C in 2005 = 345 + 815 = 1160. And sales of B is 1350. Thus, sales of B is 190 more than the combined sales and the required percentage is $\frac{190}{1160}$.

Next, 10% of 1160 is 116. And 5% will be 58. Thus, 15% of 1160 is 174. To reach to 190, we have to account for a further 190 – 174 = 16, which will be more than 1% of 1160 but less than 2%. Thus, the answer will be between 16% and 17%. Among the options, choice (b) is closest.

4. d

The percentage growth in average sales will be same as the percent growth in total sales.

Total sales of A over the period 2003-06 = 260 + 300 + 345 + 375 = 1280

Total sales of C over the period 2003-06 = 680 + 725 + 815 + 880 = 3100

Thus, sales of C is 720 + 1100 = 1820 more than A. And required percentage is $\frac{1820}{1280}$.

Since 18/12 is 150%, the required percentage will be in this vicinity and only choice (d) is close.

5. a

10% of 1465 = 146.5

5% of 1465 = 73.25

Thus, sales of B in 2007 will be 1465 + 219.75 = 1684.75

6. c

Since 37.5% = 3/8, a growth rate of 37.5% translate to a multiplying factor of $\left(1+\frac{3}{8}\right)$ i.e. $\frac{11}{8}$

$$C_{2002} \times \frac{11}{8} = 680 \Rightarrow \text{C}_{2002} = 61.8 \times 8 = 494.4$$

7. a

Sales of A in 2006 = 375 and sales of C in 2003 = 680. Thus, A's sales is less by 25 + 280 i.e. 305. And required percentage is $\frac{305}{680}$.

Half of 680 = 340 and 10% of 680 is 68. Hence the answer has to be between 40% and 50% and this is enough to narrow down to choice (a)

8. a

Sales in 2006 is 375 and sales in 2004 is 300. The ratio of the "amount" to the "principal" is 375/300 i.e. 1.25. And since 2004 to 2006 is two annual period of growth, the required growth rate is $\left(\sqrt{1.25}-1\right)\times 100$.

One might wrongly approximate the above as follows: Since square root of 1.21 is 1.1, the required growth rate is hardly any amount greater than 10%. But this is grossly wrong approximation. If you want to approximate, work as follows:

11^2 = 121 and 12^2 = 144.

Because 125 is 4 more than 121 and the total range between 11^2 and 12^2 is 23, the approximate square root of 125 (by linear approximation) will be $11+\frac{4}{23}$ i.e. approximately 11 + 0.17 = 11.18.

Square root of 1.25 will be 1.117 and the required answer will be 0.117 × 100 = 11.7. Now choice (a) appears more appropriate.

Alternate method: The total growth rate will be $\frac{75}{300}$ i.e. 25%. And the simple annual growth rate will be 12.5%. Now the CAGR will be just marginally lesser than SAGR and hence option (a) is most closest.

Exercise 2:

1. d

Income in year ending Mar '09 is 1040 and the income in previous year is 879. Thus, the required growth rate is $\frac{161}{879}$.

Next, 10% of 879 = 88; 20% will be 176. And hence 161 will be should be 15 less than 20%

Since 20% of 879 = 176, so 2% will be 17.6. Thus, 161 will be (little less than 2%) less than 20% i.e. answer has to be between 18% and 19%, answer choice (d)

2. a

Profit grew from 8.4 to 31.6 over 4 annual periods of growth. Thus, annual growth rate will be $\frac{23.2}{8.4}\times\frac{1}{4}$ i.e. $\frac{23.2}{33.6}$.

Since $\frac{1}{33.33}$ is 3, hence the required answer will be just less than 23.2 × 3 = 69.6%. Option choice (a) is pretty obvious hereafter.

3. b

$$E_{04}\times\left(1+\frac{15}{100}\right)=184 \Rightarrow E_{04}=184\times\frac{20}{23}=\frac{3680}{23}=160$$

4. c

If the growth rate from Mar '08 to Mar '09 is *r*%, then $\left(1+\frac{r}{100}\right)=\frac{11.57}{8.12}$.

And we want to find the value of $11.57\times\left(1+\frac{r}{100}\right)$ i.e. $11.57\times\frac{11.57}{8.12}$

115^2 = 13225. Thus, the required value will be marginally more than $\frac{132.25}{8.12}$.

132.25/8 = 16.53. Thus, required value will be in the vicinity of 16.5% i.e. option (c) is clear cut answer.

If choice (b), Rs. 15 cr seems too close, just cross-check: 8.12 × 15 = 121.8. But the numerator is quite greater than this. And hence answer will be greater than 15 cr.

5. c

Operating Profit = S – E.

Post the percentage increases, the operating profit will be 1.1 × S – 1.1 × E i.e. 1.1 × (S – E) i.e. the operating profit will also increase by 10%.

Thus, required answer = 107.14 + 10.7 = 117.84

6. b

The percentage increase in interest in successive years, starting from Mar '06 are $\frac{2.61}{6.85}, \frac{14.3}{9.46}, \frac{13.17}{23.74}, \frac{29.7}{37}$.

And we want to find the greatest of these.

It should be a visual check to realise that the second of this ratio is more than 100%. Thus, the required answer is mar '07 i.e. option (b)

7. c

Required percentage = $\frac{40.16}{58}$

Since 40/60 is 66.66%, the answer has to be greater than 66.66% and only choice (c) satisfies this.

8. b

NPM in the years '05 to '08 (the given options) are $\frac{8.4}{205.4}, \frac{26.2}{302}, \frac{40.2}{601}, \frac{47.5}{879}$. And we want to find the highest among them.

The first ratio is roughly 4%, second one is about 8.7%, third about 6.66% and the fourth one just over 5%. Thus, the highest among them is a visual check. And answer is Mar '06, option (b)

(This above visual check has to be done orally. For this example, the process is as given below, but you will have to work this orally yourself from here onwards.

1% of 205.4 is 2 and hence 8.4 will be roughly 4%; 26.2/300 is 8.7%; 4/6 is 66.66%; 5% of 879 is 44)

Exercise 3:

Explanations given after the eercise itself

Exercise 4:

1. b

For this question we need to find the net effect of the growth rates in year 2006, 2007 and 2008.

Also, for this question, one just has to look for only the countries given in the options.

For India (9.7%, 9.1% & 6.1%) the growth rate, on first rough cut, will be greater than 24% (simple addition of the three growth rates). Similarly for Maldives, Russian Fed and Afghanistan, the growth rate will be greater than 30%, 21% and 29%.

This rough cut approximation narrows the choice to just Maldives and Afghanistan.

For Maldives: Net effect of 19.1% increase & 6.6% increase will be more precisely 25.7 + $\frac{19\times 6}{100}$ i.e. 25.7 + 1.14 i.e. 26.84.

Next, net effect of 26.84% increase & 5.2% increase will be 32.04 + 1.3 i.e. 33.34%

For Afghanistan: Net effect of 11.2% increase and 16.2% increase will be 27.4 + 1.76 i.e. 29.16%

Next, net effect of 29.16% increase and 2.3% increase will be 31.46 + 0.6 i.e. 32%

The above are pretty precise calculations, with just the decimals being off. Thus, they are enough to find Maldives having the highest growth rate over the given period.

2. b

The growth rates that will be involved are only that of 2004 and 2005 (and NOT of 2003)

The net effect of 8.3% increase and 9.4% increase will be 8.3 + 9.4 + 0.78 = 18.48%.

Thus, the required ratio will be 118.48 : 100.

In such questions rather than reduce the ratio to match one of the options, it is far more efficient to check in which option is the first number 18% more than the second number.

Since 2/11 is 18.18%, hence 13 : 11 is closest to the found ratio.

3. d

The growth rate of which the net rate has to be found is just that of years 2007 and 2008 (and not of 2006). The net effect of 5.7% increase and 2% increase is 5.7 + 2 + 0.114 i.e. 7.814%

4. d

Since we do not know the underlying GDP figure for any of the years, we cannot comment on the values of the GDPs.

5. b

I. There are only two negative growth rates in the entire table. Thus, except for these two countries, the GDP have grown continuously for the rest of the countries. Thus this statement is true.

II. Even though there are two negative growth rates, the positive growth rates for these two countries in the rest of the years are large enough to compensate for the negative growth rate and hence over the period 2003 to 2008, even these two countries would have had a positive net growth rate in the GDP. Thus, for all countries, the GDP in 2008 will be higher than that in 2003. Thus this statement is true.

III. The GDP of USA is continuously increasing because the growth rates are positive. It is just that the growth rate are decreasing. Thus this statement is false.

6. c

The net effect of the growth rate in 2008 and 2009 should become 0% so that the GDP in 2009 is same as that in 2007.

Once could use $a + b + ab/100$ or multiplying factor.

$-2.5 + x - 0.025x = 0$ i.e. $0.975x = 2.5$ i.e. $x = 2.5/0.975$. Thus, x is marginally greater than 2.5 and the answer choice will be (c). If you are questions can it not be as great as option (d) 2.62, you could find the higher limit by just knowing that the required ratio will also be reasonably less than 2.5/0.9 i.e. 2.63.

Using multiplying factor, since 25% is ¼, hence 2.5% will be 1/40. Thus, $\left(1-\frac{1}{40}\right)\times f = 1 \Rightarrow f = \frac{40}{39}$ i.e. the percentage increase will be 1/39. Thus, the answer will be more than 1/40 and less than 1/38 i.e. more than 2.5 and less than 5.26/2 i.e. 2.63. (One should know that 1/19 is 5.26%)

7. b

The only possible country to have the GDP in 2005 lower than that in 2003 is Maldives, because for all other countries, in both the years 2004 and 2005, there was a positive growth rate.

The net effect of 9.5% increase and 9% decrease is 9.5 – 9 – 0.855 i.e. 0.5 – 0.855. This net effect is negative. And hence for Maldives, the GDP in 2005 will be less than that in 2003.

Thus, for only 1 country is the GDP in 2005 lesser than that in 2003.

8. a

GDP of Afghanistan increased by 14.5% from 2004 to 2005 and that of Maldives decreased by 9%. Thus, $\frac{1.14\times A_{04}}{0.9\times M_{04}} = \frac{5}{4} \Rightarrow \frac{A_{04}}{M_{04}} = \frac{4.5}{4.56}$

Among the given ratios only 1 : 1 best describes this ratio as the other ratios are 1 : 1.5, 1 : 1.33, 1: 1.2, all of which are far greater.

9. c

The period given is 2004 to 2008. Thus, the only growth rates in the last four columns will be relevant.

A visual check is enough for one to realise that the country with the lowest growth rates is Japan. And then the next two countries with the next lowest rates are the last two, UK and US.

Post these, the country with the next lowest rate will be Nepal and this will occupy the fourth position.

10. a

$0.91 \times M_{04} > 1.031 \times N_{04}$ i.e. M_{04} . $\Rightarrow M_{04} > \frac{103.1}{91}\times N_{04}$

Dividing directly by 91, we get $M_{04} > 1.1333 \times N_{04}$

Thus, in 2004, Maldives's GDP has to be atleast 13.33% more than that of Nepal's GD.

Exercise 5:

1. b

The population of S. Asia can be read as 56, 71, 91, 113, 136.

Thus, the growth rates will be $\frac{15}{56}, \frac{20}{71}, \frac{22}{91}, \frac{23}{113}$

1/4th of 56 is 14. Thus, 15/56 will be just over 25%.

2/7 is 28.56%

22/90 will be 24.44%

10% of 113 will be 11.3. Thus 23/113 will be just over 20%.

Hence the greatest percentage increase is 20/71 i.e. in the decade 1970-1980.

2. d

Since we do not know the population of 1950, we cannot answer the question.

3. c

For S. Asia, in Q 1 we saw that the growth rate first increased and then is decreasing continuously.

For Europe and C. Asia, since the line is straight from 1960 to 1990, the growth rate is continuously decreasing. And in 2000, the line becomes even flatter, hence even in 2000, the growth rate has decreased as compared to previous year. Thus, this region satisfies the required condition.

For S. America, the line is a straight one and hence the growth rate in successive readings is decreasing continuously. Hence this region also satisfies the required condition.

For M.E. & Africa, the population can be read as 10, 13, 17, 23, 28. Thus, the growth rates are 30%, $\frac{4}{13}, \frac{6}{17}, \frac{5}{23}$,

Now $\frac{6}{17}$ is surely more than $\frac{4}{13}$, because the numerator has increased by 50% but the denominator has not increased by such a large percentage. Hence the growth rate has increased and so this region does not satisfy the required condition.

4. b

As found in Q. 1, the highest growth rate for S. Asia was around 28.56% and in 1970-80.

In Q. 4, an even higher growth rate of 30% was seen in 1960-70. And for the same region in 1980-90, the growth rate was 6/17, which will be more than 33.33%.

Thus, looking at the option choices, we do not need to find the growth rates for the other two regions as they are not a part of the choices. Thus, answer is M.E. & Africa over the period 1980-90.

5. c

S. Asia population grew from 56 to 136, growth rate of 80/56, i.e. 10/7 i.e. 142.84%

S. America population grew from 21 to 51 i.e. a growth rate of 30/21 i.e. 10/7, same as above

M.E. & Africa population grew from 10 to 28 i.e. a growth of 180%

Europe & C. Asia population grew from 31 to 44 i.e. growth of 13/31, just about 40%.

Thus, highest growth rate of population was seen in M.E. & Africa.

6. d

This question differs from earlier one because in this question the absolute increase in population has to be greatest, whereas in earlier question the percentage growth had to be the greatest.

Needless to say, the maximum vertical distance between last point and first point is for S. Asia.

7. b

By now, we have already jotted down the growth rates for most time periods and regions.

For the years 1960-70, we have already found growth rate for S. Asia as just over 25% in Q. 1; for M.E. & Africa as 30% in Q. 3.

S. America population grew from 21 to 28 i.e. a growth rate of 7/21 i.e. 33.33%

Europe & C. Asia population increased from 31 to 35 i.e. a growth rate of 4/31, which is just over 10% and not as high as earlier found figures.

Thus, the largest growth in 1960-70 was witnessed in S. America.

8. d

From 1980 to 1990, the population of Europe and C. Asia grew from 39 to 43 i.e. a growth rate of 4/39. Since 4/40 is 10%, the total growth rate over this period was just more than 10%. But this is over an entire decade. Thus, the 'annual' growth rate will be 10%/10 i.e. 1%.

Exercise 6:

1. b

Exports to China in successive years are 252, 300, 375, 436, 426.

Thus, the growth rates are $\frac{48}{252}, \frac{75}{300}, \frac{61}{375}$ and a decrease in the last year. It should be obvious that the second growth rate is highest here, WITHOUT ANY CALCULATION.

First growth rate is lesser than 50/250 i.e. lesser than 1/5th, 20%; second growth rate is 25%; in the third growth rate, the numerator is lesser than second ratio whereas the denominator is greater than the second ratio)

2. a

Imports from Germany in successive years are 180, 267, 342, 397, 549.

The growth rates are $\frac{87}{180}, \frac{75}{267}, \frac{55}{342}, \frac{152}{397}$.

The second and third ratio are lesser than the first ratio (lesser numerators, larger denominators).

The first and last ratio also can be easily compared. Going from 1st ratio to last ratio, the numerator is lesser than double, but the denominator is more than doubled. Thus, the last ratio has to be less than the first ratio. Hence the largest ratio of these is the first ratio and imports from Germany increased by the largest percentage in 05-06.

3. c

Exports were 629 and imports were 542. Thus exports were higher by a percentage equal to $\frac{87}{542}$.

10% of 542 is 54.2; 5% is 27.1. And thus 15% will be 81.3. Hence answer has to be a little more than 15%. Looking at options, only (c) and (d) are possible.

How much more than 15%? 5.7 as a percentage of 542. And this will be just over 1%. Thus, the answer is 16%.

4. a

Total growth rate in imports from Germany, from 180 to 549, is $\frac{369}{180}$ i.e. marginally over 200%. This is total growth rate and annual growth rate will be 200/4 i.e. 50%

5. c

We need to compare $\frac{330}{209}, \frac{252}{319}, \frac{619}{315}, \frac{63}{59}, \frac{127}{180}$ and find the highest among them.

2nd ratio is lesser than 1st and ruled out. Last two are also ruled out visually. So we need to compare $\frac{330}{209}$ and $\frac{619}{315}$. The percentage increase in the numerator, from 1st to 2nd ratio, is almost 100% (lesser than that but around 90%), whereas the denominator has increased by just 50%. So the second ratio is far more than the first ratio.

Alternately, the two ratios can be thought in mixed fraction as $1\frac{121}{209}$ & $1\frac{304}{315}$ i.e. roughly 1.5 and 1.9.

6. c

Imports from UAE increased from 208 to 1059 i.e. roughly 5 times, i.e. 400% increase.

Imports from China increased from 319 to 1476 i.e. more than 4 times

Imports from USA increased from 315 to 848 i.e. a little less than thrice.

Now for the other two countries, we just have to compare and check if they have increased by more than thrice, which they have. Thus, the least growth rate is in the case of USA.

Exercise 7:

1. a

The highest compounded growth rate will be for the category which has the highest total growth rate.

Passenger vehicles increased from 11.4 to 19.5 i.e. an increase of $\frac{8.1}{11.4}$, roughly 72%

Commercial vehicles increased from 3.5 to 5.4 i.e. an increase of $\frac{1.9}{3.5}$, a little over 50%

Three Wheelers increased from 3.6 to 4.5 i.e. an increase of $\frac{0.9}{3.6} = 25\%$

Thus, passenger vehicles showed the highest growth rate.

2. d

Visually we can check that the highest growth rate will be from 05-06 to 06-07 or from 08-09 to 09-10 (the third bar is hardly any greater than second bar and the fourth bar has decreased).

The growth rate in 06-07 is $\frac{4.6-3.5}{3.5} = \frac{1.1}{3.5}$ which will be less than 33.33% and more than 30% (30% of 3.5 is 1.05)

The growth rate in 09-10 is $\frac{5.4-3.8}{3.8} = \frac{1.6}{3.8}$. This will be more than the earlier growth rate because numerator is far more (in percentage terms), whereas denominator is not so much more.

So growth rate is highest in year 2009-10.

3. c

Passenger Vehicles grew from 11.4 to 13.8, a growth of $\frac{2.4}{11.4}$ which will be less than 2.4/11 i.e. 21.8% but more than 2.4/12 i.e. 20%,

We need not find the growth rate for Commercial vehicles as the increase in them is more than that of Three Wheelers whereas the base is less than Three Wheelers. So commercial vehicles will have a growth higher than Three Wheelers.

Three vehicles grew from 3.6 to 4, a growth of $\frac{0.4}{3.6}$ i.e. 1/9 i.e. 11.11%.

Thus, Three Wheelers showed the least percentage increase.

4. b

Total vehicles sold in 2006-07 = 13.8 + 4.7 + 4 = 22.5

Total vehicles sold in 2008-09 = 15.5 + 3.8 + 3.5 = 22.8

Thus, the required growth rate is $\frac{0.3}{22.8}$.

Next, 1% of 22.8 will be 0.23 and hence the answer will be in the vicinity of 1.5% (lesser than this) i.e. option choice (b)

5. b

The total sales in 2006-07 was just found in the earlier question as 22.5. And the required percentage will be $\frac{4.7}{22.8}$.

Next 10% of 22.8 will be 2.3. Thus, the required percentage is just over 20%, choice (b)

Again if choice (c), 21.5% seems possible, we have already learnt how to eliminate two choices. Find the exact percent of a value between the two choices. 21% of 22.8 will be 2.28 + 2.28 + 0.228 i.e. 4.56 + 0.228 i.e. 4.788. Since the numerator in the required percent is less than this, the answer will be less than 21%.

6. d

Sales for 2008-09 has already been found in Q. 4 as 22.8.

Total vehicles sold in 2007-08 = 15.5 + 5 + 3.6 = 24.1

Thus, from 07-08 to 08-09, there is a decrease of 1.3 and the required percentage will be $\frac{1.3}{24.1}$.

5% of 24.1 will be 1.205

6% of 24.1 will be 1.205 + 0.24 = 1.44

Thus, 1.3 will be a decrease of approximately 5.5%.

Exercise 8:

15% of the market = 180. Thus, entire market = $180\times\frac{100}{15}$ = 1200

LG is 16.5% of the market and hence it's sales will be $\frac{16.5}{100}\times 1200 = 16.5\times 12 = 198$

Market Share of Videocon = $\frac{150}{1200}\times 100 = 12.5\%$

Since total market is 1200 and we know the sales of the three players, we can find the sales of Others as 1200 – 198 – 180 – 150 = 672. And market share of others = $\frac{672}{1200}\times 100 = 56\%$

Alternately Other's market share is going to be 100% - 16.5% - 15% - 12.5% = 56% and from this also we can find the sales.

Thus, final table will look like:

Sales of CTVs in India in 2005		
Manufacturer	Sales (Rs. Mn)	Market Share
LG	198	16.50%
Samsung	180	15%
Videocon	150	12.5%
Others	672	56%
Total Market	1200	100%

Exercise 9:

All explanations and given in the theory after the exercise.

Exercise 10:

1. a

18% of 150,600 – 15% of 125,500 = 27,108 – 18,825 = 8,283

2. c

21% of 125,500 – 15% of 125,500 = 6% of 125,500 = 7,530

3. b

Required ratio is 6% × 125,500 : 9% × 150,600 i.e. 2 × 1255 : 3 × 1506 i.e. 5 × 251 : 3 × 753 i.e. 5 : 3 × 3 i.e. 5 : 9

4. a

Required ratio is 15% × 125,500 : 18% × 150,600 i.e. 5 × 1255 : 6 × 1506 i.e. 5 × 5 × 251 : 6 × 2 × 753 i.e. 25 : 36

NOTE ON SHORT-CUT:

Solutions to all the above questions have been found in the traditional sense. In case a pie chart (shares given for two different years along with total markets in the two years), a very important figure that can reduce a lot of work is the percentage growth in the total market. While none of the question explicitly asks for this growth rate, it is advised to spend time to find it and if it turns out to be a very nice number, then your calculations get simplified very much.

The total market increased from 125,500 to 150,600 i.e. an increase percentage of $\frac{25,100}{125,500}=\frac{1}{5}$ i.e. 20%. Since this is such a nice number, all your calculations can be reduced to a large extent. Rather than use bigger numbers like 125,500 or 150,600, when one is interested in the ratio one can use 5 and 6 as substitutes for these large numbers.

Thus, in question 3, the required ratio would be 6% of 5 : 9% of 6 i.e. 5 : 9 directly.

In question 4, the required ratio would be 15% of 5 : 18% of 6 i.e. 5 × 5 : 6 × 6 i.e. 25 : 36.

How can one use the above information that market has increased by 20% to solve question like Q. 1? All market share percentages of either pie can be converted to read as market share percentages of a common base. E.g. GM's sales in 2009 is 18% of 150,600 which can be converted as 18% × 1.2 × 125,500. Thus, in Q. 1, one could also have found the answer as (18% × 1.2 – 15%) of 125,500 i.e. (21.6% - 15%) of 125,500 i.e. 6.6% of 125,500. One can find 6% as 5% + 1% i.e. 6275 + 1255 = 7530. Thus, 6.6% will be 7530 + 753 = 8283

5. d

The company with the greatest growth rate in sales will be the one with the greatest growth rate in market share. Thus, the choices, comparing the sectors of the two pies are narrowed to Hyundai (21% to 27%), GM (15% to 18%) and Tata (6% to 8%)

The percentage changes in these market shares can orally be found as 6/21 i.e. 2/7 i.e. 28.56%; 3/15 i.e. 1/5 i.e. 20%; and 2/6 i.e. 1/3 i.e. 33.33%.

Thus the sales of Tata grew by the greatest rate.

6. b

In this question, the shortcut learnt in the note is going to be very useful. We can change all percent share figures of 2009 pie to a percent figure of 125,500 by multiplying them with 1.2. Thus, we can answer the question then without any calculations. E.g.:

Increase in number of cars sold by Maruti = (31% × 1.2 – 35%) of 125,500 i.e. (37.2% - 35%) of 125,500 i.e. 2.2% of 125,500

The figure equivalent to 2.2% of above expression, for Hyundai will be 27% × 1.2 – 21% = 32.4 – 21 i.e. 11.4% of 125,500

For GM will be 18 x 1.2 – 15 i.e. 21.6 – 16 = 6.6% of 125,500

For Tata will be 8 × 1.2 – 6 = 1.2% of 125,500

For Honda or Other, we need not find it because the increase in number of cars sold by Hyundai is far too much for them to be close possibilities.

7. b

We have already seen in above question that Maruti has shown an increase of 2.2% of 125,500 in the cars sold.

Doing the same calculation for Honda, increase in cars sold = (9 × 1.2 – 11)% of 125,500, we see that the percent is a negative one and hence the number of cars sold by Honda is decreased.

Alternately since market has increased by 20% i.e. market is in ratio of 5 : 6, only if the ratio of market share is lesser than 6 : 5 i.e. lesser than 16.66%, will the sales decrease. Maruti's market share decreased by 4/35 i.e. just over 10% but less than 11.11%. So it's sales would not have decreased.

Honda's market share declined by 2/11 i.e. 18.18% and hence its sales would have declined.

8. b

In this question the sales value of the value of the entire pie (in any situation) does not matter. For any situation 100% of share will represent 360 degree i.e. 1% of share represent 3.6 degrees.

Also, it does not matter which pie we are talking about, that means, we need not find the angle separately for the two sectors (even if they are in different pies and then subtract it. All we have to do is the difference in shares of Tata in 2008 and Honda in 2009 is 6% - 9% i.e. 3% and hence the difference in the angles of these two sectors will be 3 × 3.6 is 10.8 degrees.

9. a

Market share will be 25% of 7% i.e. $\frac{1}{4}^{th}$ of 7 i.e. 1.75%

10. d

Again, in this question, we can find the answer by a very few calculations. We do not have to find the market share of Audi, all we need to know is the percentage increase in Audi's share. And it will be the percentage increase in 10% of 12 and 20% of 7 i.e. we need to the percentage increase in 1.2% and 1.4%. Thus, market share of Audi increased by 0.2/1.2 i.e. 16.66%. And the market also increased by 20%. Thus, sales increased by 16.66% + 20 + $\frac{20\times16.66}{100}$ i.e. 36.66 + 3.33 i.e. 40%. Using multiplying factors, the multiplying factor related to growth in sales will be $\frac{7}{6}\times\frac{6}{5}$ i.e. $\frac{7}{5}$, which is a percentage increase 2/5 i.e. 40%

Exercise 11:

Volume Share:

From Achiever's Academy we know that 40% of number of students = 3200 i.e. 2/5th of market = 3200. So the entire market (number of students) = 1600 × 5 = 8000

Number of students with Topper's Academy = 9.375% × 8000 i.e. $9\frac{3}{8}\%$ of 8000 i.e. 75 × 10 = 750.

Volume Market Share of Scholar Classes = 2400/8000 = 30%. And volume market share of Others = 400/8000 = 5%.

In the volume column, only Takshzila and Acumen are left. Knowing any one, we can find the other (by subtracting from the total). Hence here we should have taken help of the product price and value sales. We know both of these for Takshzila. So number of students at Takshzila

$= \frac{6 \times 10^6}{24{,}000}$ = 250. And its market share will be 250/8000 = 3.125%.

Thus students at Acumen = 8000 – (2400 + 750 + 3200 + 250 + 400) = 8000 – 7000 = 1000. And it's market share = 1000/8000 = 12.5%.

Value Share:

From Taskhzila, we know that 5% of the market is Rs. 6 mn. Hence total market is 20 × 6 = Rs. 120 mn.

Sales (in Rs. mn) of Scholar will be 30% × 120 mn = 36.

Market share of Topper's will be 9/120 = 7.5%. Market share of Others = 3/120 = 2.5%.

Since the product price of Acumen is Rs. 18000 and it has 1000 students, its sales (in Rs. mn) will be 18. And it's market share will be 18/120 = 15%

Now the only cells left are that for Others which can be found by subtracting from the total.

Product price can be found by dividing sales (in Rs. mn) by the number of students.

The final table will look like:

	Number of students		Revenue		Product Price
	#	Market Share	Rs. Mn	Market Share	
Acumen	1000	12.5%	18	15%	18000
Scholar	2400	30%	36	30%	15000
Topper's	750	9.375%	9	7.5%	12000
Achiever's	3200	40.00%	48	40%	15000
Takshzila	250	3.125%	6	5%	24000
Others	400	5%	3	2.5%	7500
Total Market	8000	100%	120	100%	15000

Exercise 12:

If the total market in value terms is Rs. x and the total market in volume terms is y units, then price per unit of each brand is (denoting x/y as k):

IBM = $\frac{17x}{14y} = 1\frac{3}{14}k$ = $1.21k$

Dell = $\frac{15x}{18y} = \frac{5}{6}k$ = $0.833k$

HP = $\frac{12x}{15y} = \frac{4}{5}k$ = $0.8k$

Sony = $\frac{18x}{11y} = 1\frac{7}{11}k$ = $1.63k$

Apple = $\frac{21x}{15y} = \frac{7}{5}k$ = $1.4k$

Toshiba = $\frac{9x}{16y}$ = $0.5625k$

Acer = $\frac{8x}{11y}$ = $0.72k$

Average Market Price of a laptop = $\frac{x}{y}$ i.e. k.

1. b

Even though we don't, know the total Value and Volume of the market, we can arrange the brands in order of their price per laptop. The order will be same as that of the ratio of their Value Market Share to their Volume Market Share, as found above. Hence the required answer is Dell

2. c

The average price at which Sony sells is $1.63k$ and that at which Apple sells is $1.4k$. Thus price of Sony is more than that of Apple by 0.23/1.4 i.e. 2.3 × 7.14% i.e. 16.422%

3. b

I. the difference is $0.833k - 0.8k = 0.033k$

II. The difference is $1.21k - 0.5625k$ i.e. approx $0.7k$

Obviously the second difference is greater, even if we don't know the value of k (remember k is positive)

4. c

All those who sell more number of laptops than Acer will have their volume market share more than Acer. Which are all the brands except Sony.

And the brands, among these narrowed down 5 brands, who have a price lesser than Acer is just Toshiba.

Thus, all other 4 brands (except Sony & Toshiba) sell more laptops than Acer and also at a higher price than Acer.

5. d

We know k = 54,000. Thus, price of HP laptop = 0.8 × 54,000 = 43,200

6. b

Any brand whose price is more than 1k has a price more than Average Market Price. Such brands are IBM, Sony and Apple.

7. a

Apple's price is $1.4k$ and Average Market Price is k. Thus Apple's price is 40% more than Average Market Price.

8. c

Average Market price is k and average price of Toshiba Laptop is $9/16k$. Thus, the average market price is more than that of Toshiba's and the percent higher is $\frac{1-\frac{9}{16}}{\frac{9}{16}} = \frac{7}{9}$

i.e. 77.77%.

Exercise 13:

Explanations given after the eercise itself

Exercise 14:

Explanations given after the eercise itself

Exercise 15:

Since $\text{Yield} = \frac{\text{Production}}{\text{Area}}$, and we are given Production and Yield figures, so re-arranging the relation to get area, we have $\text{Area} = \frac{\text{Production}}{\text{Yield}}$. With the units given, we will get the area in million hectares.

1. c

This question just requires a visual check to check if the yield of the second row of each crop is more than the yield of corresponding first row in the column related to yield in 2007.

We see that for Rice, Maize and Pulses, the yield in Rabi is more than the yield in Kharif.

2. b

The area under cultivation of rice in Kharif season in successive years can be found as $\frac{74.5}{1.91}, \frac{76.7}{1.88}, \frac{80.2}{2.12}, \frac{78.25}{2.05}$. And we have to find in which year was the growth rate the greatest.

Since we are just interested in percentage increase, we can consider the ratios as $\frac{745}{191}, \frac{767}{188}, \frac{802}{212}, \frac{782.5}{205}$. And now directly find the approximate value of them as: $4-\frac{19}{191}, 4+\frac{17}{188}, 4-\frac{46}{212}, 4-\frac{37.5}{205}$, which can then be approximated as 3.9, 4.1, 3.8, 3.85. These approximations are enough to realise that the greatest percentage increase in area is in the year 2007.

3. a

I. Area = $\frac{3.2}{0.7}$ = 4.46

II. Area = $\frac{11.44}{1.74}$ which is surely far more than 11.44/2 i.e. 5.72 but is less than 11.44/1.7 i.e. 6.7

III. Area = $\frac{8.2}{0.78}$. Approximating the denominator as 0.8, the ratio can be found to be more than 10.

Thus, increasing order will be I, II, III

4. a

The area for Rice, Jowar, Maize and Pulses is found as $\frac{76.7}{1.88}, \frac{4.6}{0.96}, \frac{10.4}{1.76}, \frac{4.7}{0.45}$

Obviously the first ratio is in the vicinity of 38 to 40, whereas all of the others are just in the range of 5 to 10. So area under cultivation of Rice is the greatest.

5. a

Area under cultivation of Jowar in Kharif season in successive years is $\frac{4.18}{1.03}, \frac{4.6}{0.96}, \frac{4.88}{1.11}, \frac{5.12}{0.9}$. And we want to find the least of them.

The last ratio is the largest (greatest numerator and least denominator). Also the second ratio is more than the first ratio (for same reason, larger numerator and lesser denominator). So the comparison is just between $\frac{418}{103}, \frac{488}{111}$. The ratios can be expressed in mixed fraction as $4\frac{6}{103}, 4\frac{44}{111}$. This is enough to realise that the second of these is greater. Hence the required least area is in the year 2006.

Exercise 16:

Denoting the ratio of weight to height as R, we know that R = $\frac{\text{Weight}}{\text{Height}}$ and we also know the weights from the second graph. Thus, the heights can be found as Height = $\frac{\text{Weight}}{\text{R}}$

1. c

Height of Samit when he was 10 years old = 42/0.8 = 52.5 inches.

Height of Romit when he was 10 years old = 60/1.1 = 54.54 inches.

Thus, the required difference = 54.54 – 52.5 = 2.04 inches

2. d

Arpit's height when he was 6 years old = 40/1.3 = 30.76

Arpit's height when he was 10 years old = 46/1 = 46

Thus, he grew by 15.24 inches.

3. a

The heights of Amit, Samit, Arpit and Romit when they were 8 years old are $\frac{48}{1.3}, \frac{46}{1.2}, \frac{38}{1}, \frac{54}{1}$ i.e. 36.9, 38.33, 38, 54. Thus the shortest among them is Amit.

4. c

The heights of Amit, Samit, Arpit and Romit when they were 4 years old are $\frac{40}{1.3}, \frac{30}{1}, \frac{34}{1.5}, \frac{36}{1.4}$ i.e. 30.76, 30, 22.66, 25.71

The heights of Amit, Samit, Arpit and Romit when they were 16 years old are $\frac{76}{1.2}, \frac{60}{0.9}, \frac{52}{1}, \frac{74}{1.1}$ i.e. 63.33, 66.66, 52, 67.27

Thus, the increase in their heights from 4 years to 16 years was 32.57, 36.66, 29.33, 41.56. Thus, Arpit grew the least.

5. b

We need to find the percentage increase from $\frac{32}{1.3}$ to $\frac{54}{1.3}$ which will be the same as the percentage increase from 32 to 54 i.e. 22/32 i.e. 11/16 i.e. 8/16 + 3/16 i.e. 50% + 6.25% × 3 = 68.75%

6. b

I. The ratio of height to weight is the reciprocal of the values of the first line graph. Since the ratio of weight to height increased for each of Romit, Arpit and Samit atleast once during the years given, the ratio of the height to weight decreased atleast once for them. So this statement is false.

II. The ratio of weight to height of Romit is a straight line. This however does not mean that the percentage decrease in the ratio is same for every two years. Infact, the percentage decrease in the ratio is increasing over this time period (because of same difference but reducing base). Hence this statement is also false.

III. Arpit's height every two years is $\frac{34}{1.5}, \frac{40}{1.3}, \frac{38}{1}, \frac{46}{1}, \frac{52}{1.1}, \frac{56}{1.1}, \frac{52}{1}$ i.e. 22.66, 30.76, 38, 46, 47.27, 50.9, 52 i.e. the height is continuously increasing. Hence this statement is true.

Exercise 17:

Population Density = $\frac{\text{Poulation}}{\text{Area}}$. Thus, Population of a country can be found by multiplying the population density to the area.

1. c

Since for all countries, the area remains same over the two years, 2002 and 2008, then if population density has increased from 2002 to 2008, population has increased and if population density has decreased, then population has decreased. The number of countries having their population density in 2008 more than that in 2002 is 6 (except Bulgaria and Ukraine)

2. a

The increase in population of Myanmar = (70 – 62) × 680,000 i.e. 8 × 680,000 = 5,440,000 i.e. 5.44 mn.

3. b

Since the area has remained the same in 2002 and 2008, the percentage increase in the population is same as the percentage increase in the population density. Thus, the required percentage growth is percentage growth from 56 to 46 i.e. a decrease of 10/56 i.e. (10/14)/4 i.e. 71.4%/4 i.e. 17.8%

4. c

The country with the largest population is going to be one that has a higher population density as well as a higher area. Looking for this combination in the graph, the choices are narrowed down to Ukraine and Myanmar. (Each of the rest of the countries has both the area and the population density lower than one of these two countries)

Population of Ukraine in 2008 = 76 × 600 = 45600 thousand.

Population of Myanmar in 2008 = 70 × 680 = 47600 thousand.

Thus, in 2008, the highest population is of Myanmar.

5. c

Area per person is the reciprocal of population density. So it will be the highest for the country which has the lowest population density. In the year 2002, the country is Nicaragua.

6. a

Since area remains the same, population density will also increase by 10%. Thus, required population density = 68 + 6.8 = 74.8

7. c

The difference in the population can be found by (difference in density) × (area).

Thus, again we should search for a combination where both the area and the difference in the population density is high. Thus, the answer should be an obvious choice, Myanmar, since it has the largest area and the difference in the population density is also one of the largest (Iraq and Burkina Faso has a very slightly higher difference in population density, but their areas are far lower than Myanmar).

8. b

The percentage growth in the population is same as the percentage growth in the population density (because area is constant)

Thus, we need to search for the largest percentage increase in population density. This will be for a country where the difference between population density is large (with that in 2008 being higher than 2002) and at the same time the base, density in 2002, should be lower. Again a visual glance can ascertain that fortunately both these conditions exist simultaneously for Burkina Faso. Thus, percentage increase in population will be largest for Burkina Faso.

Exercise 18:

1. d

Since we do not know the actual GDP values for any of the country, we cannot have any idea of the value of the cash deficit.

2. c

The growth rate of the cash deficit is the net effect of the growth rate in 'deficit as a percentage of GDP' and the growth rate of the GDP [because Cash deficit = (Deficit as a % of GDP) × (GDP)]

Looking at the options ...

India: Growth rate of GDP in 2005 is 9.4% and the growth rate in 'deficit as a percentage of GDP' is almost negligible (from 3.25 to 3.3 i.e. a percentage increase of 0.05/3.25 i.e. 5/325 i.e. 1/65 i.e. approx 1.5%). Thus, the growth rate in deficit will be a little more than 9.4%

Czech Republic: Growth rate of GDP in 2005 is 6.2% and the growth rate in 'deficit as a percentage of GDP' (from 3.25 to 3.5) is 0.25/3.25 i.e. 1/13 i.e. 7.7%. Thus, the growth rate in deficit will be 6.2 + 7.7 + (6.2 × 7.7)/100 = 13.9 + 0.4 = 14.3%

Egypt: Growth rate of GDP in 2005 is 4.6% and the growth rate in 'deficit as a percentage of GDP' (from 5.5 to 6.5) is 1/5.5 i.e. approximately 18%. Thus, the growth rate in deficit will be 4.6 + 18 + 0.8 = 23.4%

Thus, the largest growth rate in deficit will be for Egypt.

3. b

If GDP of Spain in 2005 is x, then the required ratio is

$$\frac{1.25\% \times x}{2.5\% \times (1.04 \times 1.038 \times x)}$$

Rather than multiply 1.04 × 1.038, let's use $a + b + ab/100$ to find the net effect of 4% increase and 3.8% increase. It will be 4 + 3.8 + 0.152 = 7.95%, say approx 8%.

Thus, the required ratio will be $\frac{1}{2 \times 1.08}$ i.e. 1 : 2.16. The closest option choice is (b)

4. d

Since the ratio of GDP's of Poland and Czech are not known for any of the years, the ratio cannot be determined.

5. b

If the GDP of India and Poland in 2006 is x and y respectively, then as per the data,

$$\frac{1\% \times (1.09 \times x)}{2\% \times (1.068 \times y)} = \frac{5}{4} \Rightarrow \frac{x}{y} = \frac{5}{2} \times \frac{1.068}{1.09}$$

Since the second ratio is almost 1, the ratio will be just lower than 5 : 2. The only option close enough to this is 5 : 2. The rest of the options are way off the mark.

6. c

Using the same process as used in Q. 2 ...

In 2008, Egypt's GDP grew by 7.2%. and the percentage growth in 'deficit as a percentage of GDP' from 2007 to 2008 (from 4.5 to 6.5) is 2/4.5 i.e. 4/9 i.e. 44.44%.

Thus, the growth rate in the deficit = 44.44 + 7.2 + (44.44 × 7.2)/100 i.e. 51.64 + approx 3 i.e. 54.64%

7. d

I. From 2003 to 2004, the percentage decrease in 'deficit as a percentage of GDP' is very less as compared to the growth rate in GDP in 2004. Thus, the net effect of the two percentages will be positive implying that India's deficit in 2004 is more than that in 2003. Thus, this statement is false.

II. The growth rate of Egypt's GDP can be read directly from the second graph and we see that growth rate in GDP is 3.2%, 4%, 4.4%, ... Thus the growth rate of Egypt's GDP is also increasing. Hence this statement is also false.

8. c

The 'deficit as a percentage of GDP' in successive years from 2003 is 5%, 3.25%, 3.5%, 4.25%, 1.75%, 1.5%. Thus the growth rate in 'deficit as a percentage of GDP', in successive years from 2004 is –1.75/5; 0.25/3.25; 0.75/3.5; –2.5/4.25; and –0.25/1.75 i.e. –35%; 1/13 i.e. 7.7%; 21.4%; –58%; and –14.28%

And the growth rate in the GDP in successive years from 2004 is 4.6%, 6.2%, 6.8%, 6% and 2.45

Thus, in 2004, the growth rate in fiscal deficit would have been the net effect of –35% and 4.6%, which would be negative i.e. the fiscal deficit would have decreased by around –31%. Thus, deficit in 2004 is quite lower than the deficit in 2003.

In 2005, the growth in fiscal deficit would have been the net effect of 7.7% and 6.2% i.e. the fiscal deficit would have grown by around 14.3%. However this growth is not enough for the deficit to cross the 2003 levels.

In 2006, the growth in fiscal deficit would have been the net effect of 21.4% and 6.8% i.e. the fiscal deficit would have grown by around 29%. This and the last year's growth is enough for the fiscal deficit in 2006 to be more than the fiscal deficit of 2003. Thus, the highest fiscal deficit so far is in the year 2006.

Post this, the growth in fiscal deficit is always negative (because the growth rate in 'deficit as a percentage of GDP' is a large negative value that easily overshadows the growth in GDP) and hence the highest fiscal deficit is in the year 2006.

Exercise 19:

We have been given two ratios, $\frac{\text{Arable Land}}{\text{Total Land}}$ and $\frac{\text{Arable Land}}{\text{Population}}$. Thus the ratio of the first measure and the second measure will us $\frac{\text{Population}}{\text{Total Land}}$.

1. b

Since the total area of a country remains constant, the percentage increase in the arable land will be the same as the percentage increase in 'arable land as a % of total land'. Thus, the required growth rate is the percentage increase from 12.3 to 13.10 i.e. 0.8/12.3. Since 8/12 is 66.66, the ratio 8/123 will be just less than 6.66%. Closest choice is (b)

2. c

Required population density = $\frac{0.113}{0.3}$ = 0.37

3. a

Population density of country C in 1997, 2002 an 2007 is given by $\frac{17.6\%}{0.27}, \frac{19.5\%}{0.26}, \frac{20\%}{0.27}$. The first ratio is the least of these and hence population density is least in the year 1997.

4. a

Population density of A, B, C, D and E in the year 2007 is $\frac{13.1}{0.3}, \frac{24.4}{0.32}, \frac{20}{0.27}, \frac{28.4}{0.4}, \frac{19}{0.35}$ (since have to compare them, the percent sign of numerator is dropped). And we want to find the highest of these.

Very rough calculations results in the ratios being 43, 80, almost midway of 66.66 and 80, 71, between 63 and 47. Thus, the largest of them can easily be found as the second ratio i.e. for country B.

5. c

Since the total area of a country remains constant, the percentage increase in the population of a country is same as the percentage increase in the population density of a country. And the population density of E in 1997 is $\frac{13.7\%}{0.35}$ and in 2007 is $\frac{19\%}{0.35}$. Thus, the percentage increase in this is same as the percentage increase in 13.7 to 19 i.e. 5.3/13.7

10% of 13.7 is 1.37. Hence 5.3 will be just less than 40%. This is enough to identify the answer as choice (c)

Exercise 20:

1. b

I. Either of the two teams may have batted first and nothing from the data suggests which team batted first. So this question cannot be answered.

II. At the end of 20 overs, the run-rate of New-Zealand was more than that of Sri Lanka. This means that at end of 20 overs, New Zealand has scored more than Sri Lanka and hence has won the match.

2. c

Sri Lanka at end of 10^{th} over had a score of 5.4 × 10 = 54 runs. And at the end of 20 overs had a score of 6.75 ×20 = 135. Thus runs scored from 11^{th} to 20^{th} overs = 135 – 54 = 81.

3. a

It will be cumbersome to find the over by checking each over. So one should have checked only for the options that are given.

Score in 17^{th} over = Score at end of 17^{th} over – Score at end of 16^{th} over = 6.5 × 17 – 5.9 × 16 = 110.5 - 94.4 i.e. 110 – 95 = 15

NOTE: When the number of runs scored (a natural number) is divided by 17 to get the run-rate, it will be a decimal value, the exact value of which is difficult to be read from the graph. Thus, we have to intelligently convert the runs scored to a natural number. The run rate at end of 17^{th} over is not exactly 6.5 but is less than it. Hence 110.5 is converted to 110. The run-rate at end of 16 overs is not exactly 5.9, but more than it. Hence runs scored is converted from 94.4 to 95.

Using a similar process, runs scored in 19^{th} over = 6.6 × 19 – 6.4 × 18 i.e. 125.4 - 115.2 i.e. 126 – 116 = 10.

Runs scored in 20^{th} over = 6.95 × 20 – 126 = 139 – 126 = 13.

Thus, highest runs are scored in the 17^{th} over.

It is very difficult to arrive at the answer by a visual check of the graph.

4. d

Starting from value-wise the second option i.e. over 8, runs scored at end of 8 overs = 5.5 × 8 = 44.

Runs scored at end of 9^{th} over = 5.3 × 9 = 47.7 i.e. 48

Runs scored at end of 10^{th} over = 5.4 × 10 = 54. Thus, the first 50 runs were scored in the 10^{th} over.

5. b

For each over, whosever graph is higher, has a higher score at the end of that over. Thus, from the 2^{nd} to the 20^{th} over, line graph of New Zealand was above that of Sri Lanka for overs 5 to 13 and 20 i.e. 10 over in all.

6. b

Again working with options:

Difference in runs at end of 8th over = (7 – 5.5) × 8 = 1.5 × 8 = 12 runs

Difference in runs at end of 9th over = (7.2 – 5.3) × 9 = 1.9 × 9 = 17.1 i.e. 17 runs.

Difference in runs at end of 10th over = (6.7 – 5.4) × 8 = 1.3 × 10 = 13 runs

Difference in runs at end of 16th over = (6.6 – 5.9) × 16 = 0.7 ×16 = 11.2 i.e. 11 runs.

The maximum difference was at the end of 9th over.

Exercise 21:

Let the proportionality factor for revenue be n and that for Quantity be m. Thus actual revenues can be taken as $100n$, $115n$, $135n$, ... and actual quantity be taken as $100m$, $112m$, $120m$, ...

Also let's denote n/m as k, since revenue/quantity i.e. price per unit will surely be present in the questions.

1. b

The price per car in successive years from 2000 was k, $1\frac{3}{112}k$, $1\frac{1}{8}k$, $1\frac{1}{15}k$, k, $0.95k$. Thus the highest is for the year 2002.

2. d

Rather than finding the percentage increase in revenues and in number of cars, since we have already found the price per car in the earlier question, it is easier to work with the logic that whenever the price per car has increased from the previous year, the percentage increase in revenue is more than the percentage increase in number of cars.

Price per car is more than the previous year in the years 2001 and 2002 only. Thus, only in 2 years did revenue increase at a higher rate than quantity.

3. b

The average price decreased from k to $0.95k$. This is percentage decrease of 5%

4. b

The average price in 2003 was $1\frac{1}{15}k$. And the price per car was more than this only in 2002.

5. c

$160n = 120$ i.e. $n = 3/4$

$100m = 1900$ i.e. $m = 19$

Thus, $k = \frac{n}{m} = \frac{3}{4\times19}$. And the price in 2005 = $\frac{19}{20}\times\frac{3}{4\times19} = \frac{3}{80}$ crore i.e. 0.0375 cr i.e. 3.75 lac.

Exercise 22:

Let the proportionality factor of sales be n and that of expenses be m. Thus, actual sales values can be taken as $100n$, $95n$, $100n$, $118n$, ... and the actual expenses can be taken as $90m$, $108m$, $100m$, $110m$, $100m$, ...

1. d

Since no relation of n and m is known, the question cannot be answered.

2. d

$130n - 100m = 30$ lac i.e. $13n - 10m = 3$.

And we want to find the value of $142n - 90m$.

Since there is no unique solution for n and m and also because $142n - 90m$ is not a multiple of $13n - 10m$, we cannot ascertain he required value uniquely. Hence cannot be determined.

3. b

Since sales and expense in 2000 were equal, $100n = 90m$ i.e. $n = 0.9m$

In 2008 sales was $175n$ i.e. $175 \times 0.9m$ i.e. $157.5m$ and expenses were $140m$. Thus sales was greater than expenses by 17.5/140 i.e. a little more than 10%. How much more? 3.5/140 i.e. 0.5/20 i.e. 2.5%. Thus, required percentage is 12.5%

4. c

I. Just because the indices of sales and expenses are same in 2002, does not mean sales and expenses are equal. So this statement need not be necessarily true.

II. Since the index number of sales in 2000 and 2002 is the same, the sales has to be the same. Remember the index numbers mirrors the underlying values exactly as regards percentage increase or ratios. Hence this statement is necessarily true.

III. Index numbers mirrors the underlying values exactly as regards percentage increase. Thus percentage increase in sales, from 100 to 175 is 75%. And percentage increase in expenses, from 90 to 140 is 50/90 i.e. 55.55%. Hence this statement is also necessarily true.

5. a

Since there is a profit in 2001, $95n > 108m$ i.e. n is greater than m. In the graph we see than the index for Sales is always more than the index of expenses except in 2001, when they are equal. Thus, the actual sales values will always be more than the actual costs, since $n > m$. Hence the company will show profits in all of the 9 years.

6. a

Profit in 2000 = $100n - 90m$. Since this is a profit, $100n - 90m > 0$ i.e. $100n > 90m$ i.e. $n > 0.9m$

Now for every year we have to compare the profit with the above. So in each of the expression for profit, we will substitute the limiting value of n as 0.9m and keep in mind that n is greater than this.

For 2001: $(95n - 108m)$? $(100n - 90m)$. Transposing will not affect the inequality and hence $-18m$? $5n$. Obviously for positive m and n, the LHS is lesser than the RHS. Hence profit in 2001 is lesser than that in 2000. This was also possible to be deduced visually from graph.

For 2002: $(100n - 100m)$? $(100n - 90m)$ i.e. $9m$? $10m$. Obviously the LHS is lesser i.e. profit in 2002 is lesser than that in 2000.

For 2003: $(118n - 110m)$? $(100n - 90m)$ i.e. $18n$? $20m$. Now either of the sides could be greater. Thus, for this year, we cannot conclude if the profit will be more or less.

For 2004: $(130n - 100m)$? $(100n - 90m)$ i.e. 30n ? $10m$. Since n is definitely greater than $0.9m$, the LHS is greater and profits in this year are greater than that in 2000.

For all further years, a similar situation will occur and hence except for 2001, 2002 and 2003, all other years will definitely have a profit more than that in year 2000.

7. d

Comparing the profit in two years, say 2005 and 2006, because the distance between the two graphs is maximum for these years,

$(142n - 90m)$? $(150n - 96m)$ i.e. $6m$? $8n$

One will not be able to comment on which of the two is greater. It would depend on the relative values of m and n. Hence once cannot find the year when maximum profits are made.

If you are not convinced by the above, look the following two situations which satisfy all the requirements of compatibility with the index numbers and yet maximum profits are made in different years.

If $n = m = 1$, then

	'00	'01	'02	'03	'04	'05	'06	'07	'08
Sales	100	95	100	118	130	142	150	160	175
Expenses	90	108	100	110	100	90	96	120	140
Profit	10	-13	0	8	30	52	54	40	35

If $n = 2$ and $m = 3$, then

	'00	'01	'02	'03	'04	'05	'06	'07	'08
Sales	200	190	200	236	260	284	300	320	350
Expenses	270	324	300	330	300	270	288	360	420
Profit	-70	-134	-100	-94	-40	14	12	-40	-70

8. a

Profit percentage will be directly proportional to the ratio of sales to expenses. Thus, even if we do not know the value of m and n, yet the ratio m/n will be present in the ratio of sales to expenses for each of the year. And since we just have to compare, it will not affect our answer.

Rather than find the ratio for all the years, just find the ratio for the years given in the options. The ratio of sales to expenses for option (a), (b) and (c) are $\frac{142}{90}, \frac{150}{96}, \frac{160}{120}$.

And we want to find the highest among them. The first and last ratio by actual division is 1.5777 and 1.33. The second ratio is 50/32 i.e. 25/16 i.e. 25 × 0.0625 = 1.5625. Thus the greatest ratio is for option (a).

Exercise 23:

1. b

M-800 is 19% of 45% of 18000 = 19 × 45 × 1.8 = 19 × 81 = 1539

2. a

Maruti Omni = $\frac{5}{9} \times \frac{1}{100} \times \frac{9}{20} \times 18000 = 45$

3. a

The required ratio = $\frac{32\% \times 45\%}{12.5\%} = \frac{32 \times 45}{12.5 \times 100} = \frac{144}{125}$

4. c

Maruti currently sells 45% of 18,000 = (50% - 5%) of 18,000 = 9000 – 900 = 8,100

The total market would now become 18,900 and Maruti will become 9,000. Thus, market share of Maruti = 90/189 i.e. 47.6%

5. c

Currently Maruti = 8100 and Swift = 24% of 8100 = 1944

Let the increase in number of swifts be x. Since 30.9090..% is $30\frac{10}{11}$% i.e. $\frac{340}{1100}$ i.e. $\frac{17}{55}$, we have $\frac{1944 + x}{8100 + x} = \frac{17}{55}$.

Solving this we can get x.

If you want to avoid calculations, you could also assume sales of Maruti as 100 and at end apply unitary method. With this, we would get $\frac{24 + x}{100 + x} = \frac{17}{55}$ i.e. $1320 + 55x = 1700 + 17x$ i.e. $38x = 380$ i.e. $x = 10$.

But since sales of Maruti is not 100 and is 8,100, the increase will be 10 × 81 = 810

Exercise 24:

The numbers given are degree that the sector makes at the center. Since 360 degrees = 100%, to change degrees to percentages, we will have to divide by 3.6. One need not do this calculation unless absolutely must. Read through the explanation to see how one can avoid this.

Assume the income in first half year as $360x$ and in second half year as $360y$.

1. c

$24x = 30y$ i.e. $\frac{x}{y} = \frac{5}{4}$. This will be the ratio of the income in the two half years.

2. c

The ratio of the expenses in the two half years will be 60 : 40 i.e. 3 : 2. Thus, $\frac{228x}{210y} = \frac{3}{2} \Rightarrow \frac{x}{y} = \frac{105}{76}$.

And we need to find $\frac{30x}{20y}$ i.e. $\frac{3}{2} \times \frac{105}{76} = \frac{315}{152}$

The closest option is 2 : 1

3. b

Once could use ($18x + 15y$) = 4.66% ($360x + 360y$) to find the ratio of x and y and then proceed to find the ratio ($24x + 30y$)/($360x + 360y$).

An easier way is to realise use Alligation to find the ratio x/y.

4.66% is $4\frac{2}{3}$ i.e. 14/3%. This in degree terms is $\frac{14}{3} \times 3.6$ = 16.8 degrees.

Thus, the weighted average of 18 and 15 is 16.8. Hence the weights are in the ratio 1.8 : 1.2 i.e. 3 : 2

Now we want to find the weighted average of 24 degrees and 30 degrees with weights being 3 : 2. The average is thus (72 + 60)/5 i.e. 132/5 degrees i.e. 26.4 degrees. In percentage terms this will be 264/36 i.e. 22/3 = 7.33%

For questions 4 & 5:

$360x = 120$ i.e. $x = 1/3$ and $360y = 144$ i.e. $y = 2/5$.

4. d

Expense in first half = 228/3 = 76, Net profit in first half = 60/3 = 20

Expense in second half = 210 × 2/5 = 84, Net profit in second half = 85 × 2/5 = 34.

Thus total profit = 54 and total expenses = 160. Thus, required percentage = 54/160 i.e. 33.75%

5. a

Interest paid in first half = 30/3 = 10. Interest paid in second half = 20 × 2/5 = 8.

Thus, required percentage 25% more.

Exercise 25:

Let the total number of employees of the company be $100x$.

1. d

All of $8x$ employees who have a doctorate degree belong to the group of $10x$ who are are more than 50 years old. Thus, the required percentage is $2x/10x$ i.e. 20%

2. c

If group of matriculates is denoted by A and group of those less than 30 years is denoted by B, we need to find $A \cup B$ and it is given by $A + B - A \cap B$ = $12x + 15x - 10x = 17x$, which are 17% of total number of employees.

3. a

The total number of employees who are more than 40 years old is $40x$. And of this the number of matriculates, graduates, post-graduates and doctorates are $4x$, $20x$, $12x$ and $4x$.

The total number of matriculates, graduates, post-graduates and doctorates are $12x$, $48x$, $32x$ and $8x$.

Thus, the number of matriculates, graduates, post-graduates and doctorates among those who are less than 40 years of age are $8x$, $28x$, $20x$ and $4x$. Thus, the required ratio is 2 : 7 : 5 : 1

4. b

Total number of graduates is $48x$. And of these those less than 30 years, 31 to 40, 41 to 50 and above 50 years are $6x$, $24x$, $12x$ and $6x$. These account for all the graduates. So any one not among these is not a graduate.

The total number of people in the age group 31-40 is $45x$. And $24x$ of them are graduates. Thus, required percentage = 24/45 i.e. 8/15 i.e. 53.33%

5. d

Graduates or Post-graduates together account for $80x$ people. Whereas those aged from 31 to 50 years old are only $75x$. Thus, $5x$ employees that are graduates or post-graduates have to be less than 30 years old or more than 50 years old. However 5% is not the answer. 5% refer to the persons we need, but it is a percentage of the total employees. Whereas the question asks us percentage of those who are graduates or post-graduates. Thus, the answer is $5x/80x$ i.e. 6.25%.

Exercise 26:

1. a

If the batch strength in 2008 and 2009 is x and y respectively, then $\frac{24\% \times y}{30\% \times x} = \frac{8}{9} \Rightarrow \frac{y}{x} = \frac{10}{9}$ i.e. total strength increased by 11.11%

2. b

The greatest percentage decrease in the share is in Finance. At the same time, Finance also had the largest number of students opting for it in 2008. Thus, both these facts occurring together implies that the decrease in number of students is maximum for Finance.

3. d

The functional area that showed the maximum percentage increase in the number of students opting for it is the functional area that showed the greatest increase in it's share i.e. Operations.

Thus, we have 12% of y – 8% of x = 20, where x and y are the batch strength in 2008 and 2009. However this information is not enough to find x or y. So we cannot determine the information needed.

4. c

Since the share of students not opting for Systems in 2008 and 2009 is the same, the percentage change in them is the same as the percentage in the strength of 2008 and 2009. And this will also be the percentage change in the number of opting for Systems (again because the share of students opting for systems is the same in both the years. Thus, the required answer is 10% itself.

Alternately, If the batch strength in 2008 and 2009 is x and y respectively, then $\frac{80\% \times y}{80\% \times x} = \frac{11}{10} \Rightarrow \frac{y}{x} = \frac{11}{10}$. And we need to find $\frac{20\% \times y}{20\% \times x}$, which will also be 11/10 i.e. 10% increase.

5. c

If the batch strength in 2008 and 2009 is x and y respectively, then $\frac{8\% \times x}{24\% \times y} = \frac{2}{7} \Rightarrow \frac{x}{y} = \frac{6}{7}$.

And we need to find $\frac{28\% \times x}{19\% \times y}$ i.e. $\frac{28}{19} \times \frac{6}{7} = \frac{24}{19}$

Exercise 27:

1. b

25% of Maharashtra's oil-seed production = 10% of (30% of India's oil-seed production).

Thus, Maharashtra's oilseed production = 3/25 of India's oilseed production. Thus, required percentage = 12%

2. c

Since mustard is $1/5^{th}$ of total oilseed production of Maharashtra, hence total oilseed production of Maharashtra = 7.5 mn tons. Thus, required percentage = 7.5/80 i.e. 9.375%

3. b

Total production of sunflower in entire India = 12.5% of 80 = 10. This production is divided between Maharashtra and Rest of India in the ratio 1 : 4 i.e. Maharashtra produces 2 mn tons of sunflower. Hence it produces $2 \times \frac{100}{15}$ = 13.33 tons of oilseed. Thus, Rest of India produces 80 – 13.33 = 66.66 tons of oilseeds of which sunflower is 8 mn tons. Thus, required percentage is 8/66.66 i.e. 8 × 3/2 = 12%

4. b

Ratio of soyabean production in Maharashtra and Rest of India is 3 : 5 (from 40% less). Also total soyabean production of India is 20% of 80 = 16. Thus, soyabean production in Maharashtra is 6 mn tons and this is 40% of its total oilseed production. We need to find 25% of Maharshtra's production. Since 40% is 6, 25% will be $6 \times \frac{5}{8}$ = 3.75 mn tons.

5. b

Let Maharashtra's total oilseed production be x. Since Maharashtra's production for each variety of oilseed has to be less than the all India's production of that variety, hence we have,

For groundnut: 25% of $x \leq 24$ i.e. $x \leq 96$

For mustard: 20% of $x \leq 30$ i.e. $x \leq 150$

For soyabean: 40% of $x \leq 16$ i.e. $x \leq 40$

For sunflower: 15% of $x \leq 10$ i.e. $x \leq 66.66$

Since all the conditions have to be met simultaneously, the limiting condition is that $x \leq 40$. Thus, maximum production of Maharashtra can be only 40 mn tons.

Exercise 28:

1. a

In the field of medicine, the ratio of patents granted to the number applied, for successive years from 2002-03 are $\frac{300}{950}, \frac{450}{2500}, \frac{200}{2300}, \frac{450}{2200}, \frac{800}{3200}$. And we want to find the highest among them. The last ratio is very easy to reduce to ¼ i.e. 25%. So now we have a bench mark to compare with and we see that in each of the second, third or fourth ratio, the denominator is more than 4 times the numerator. So all these are lesser than 25% and rule out. But in the first ratio, the denominator is less than 4 times the numerator. Hence the first ratio is more than 25% (visually also one can check that the first ratio is more than 30%). Hence the required year is 2002-03

2. d

In 2005- 06 …

Chemical, applied 3200, granted 1150 i.e. more than 30%

Medicine, applied 2200, granted 450 i.e. a little more than 20%

Electrical, applied 1300, granted 450 i.e. more than 30%

Mechanical, applied 3150, granted 1450 i.e. more than 40%

Thus, without the need of precise calculations the answer can be found as Mechanical.

3. a

Looking at the number of patents applied in field of Chemical, the largest absolute increase in numbers is from 2002-03 to 2004-04 and also this is on the lowest base. Hence the largest growth rate in number of applications is in the year 2003-04.

4. c

Rather than find for all the years & field combination, just work with the options. For the four choices given, the ratio of granted to applied respectively are $\frac{450}{2500}, \frac{400}{2100}, \frac{200}{1250}, \frac{400}{2300}$. And we want to find the least among them.

The second and fourth ratio has twice the numerator as compared to the third ratio. However their denominators are less than twice the denominator of the second ratio. Thus, the second ratio is the least among the last three ratios. Comparing the third ratio with the first, denominator of first is twice the denominator of third, but numerator of first is more than twice the numerator of third. Thus, first ratio is more than the third. And hence the third ratio, third option choice, has the least required percentage.

5. d

A visual check of the relative sizes of the first two bars in the granted group of each filed reveals that for both Electrical and Mechanical the second bar is more than double the first bar, which is not the case with Chemical and Medicine. So our answer has to be either Electrical or Mechanical.

From 2002-03 to 2003-04,

patents granted in Electrical grew from 150 to 400, i.e. a increase of 166.66%

patents granted in Mechanical grew from 200 to 550, i.e. a increase of 175%

Thus, highest percentage increase is for the field of Mechanical.

Exercise 29:

Fiscal Deficit as a % of GDP = $\frac{\text{Fiscal Deficit}}{\text{GDP}}$. Hence GDP = $\frac{\text{Fiscal Deficit}}{\text{Deficit as a \% of GDP}}$

1. d

For high GDP, the Fiscal Deficit should be high and it as a percentage of GDP should be low. Both these occur simultaneously for the year 07-08 (highest fiscal deficit, lowest percentage of GDP) and hence without any calculation one can identify the GDP to be the highest in 07-08.

2. a

Since more questions might need the GDP values, let's find them approximately.

The GDP in successive years from 02-03 are $\frac{145}{5.9\%}, \frac{120}{4.5\%}, \frac{125}{4\%}, \frac{145}{4.1\%}, \frac{140}{3.5\%}, \frac{150}{2.7\%}$

Assuming 5.9% as 6%, the GDP in 02-03 will be more than 2416.6

GDP in 03-04, 04-05 can be directly found by division as 2666.6 and 3125 respectively.

Assuming 4.1% as 4%, the GDP in 05-06 will be less than 3625

GDP in 06-07, 07-08 can be directly found by division as 4000 and 5555 respectively.

Thus, the percentage increase from 02-03 to 03-04 will be the least.

3. c

We know that $\frac{FD_{07-08}}{GDP_{07-08}} = 2.7\%$

$$\frac{FD_{08-09}}{GDP_{08-09}} = \frac{1.1 \times FD_{07-08}}{1.05 \times GDP_{07-08}} = \frac{22}{21} \times 2.7\% = \frac{59.4}{21} = 2.83$$

4. b

The deficit that is more than the targeted one is 0.6% of the GDP.

We know that 4.1% of GDP = 145. Thus, 0.6% of the GDP will be $145 \times \frac{0.6}{4.1}$ = 35.3 × 0.6 = 21.18

5. c

As found in Q. 2, GDP is continuously increasing in all the years.

As compared to previous year Fiscal Deficit has increased in 04-05, 05-06 and 07-08.

Of these years, fiscal deficit as a % of GDP decreased, as compared to previous year, in 04-05 and 07-08.

Thus, all the given conditions were achieved in only two years.

Exercise 30:

One need not make a table like this, but one could write the values adjacent to the bars on the graph itself.

	83-84	84-85	85-86	86-87	Total
Mkt Borrowings	4	6.4 – 4 = 2.4	9.2 – 6.4 = 2.8	12.8 – 9.2 = 3.6	12.8
Small Saving	2.4	6 – 2.4 = 3.6	10.4 – 6 = 4.4	13.6 – 10.4 = 3.2	13.6
Rec. of loans	2.8	5.2 – 2.8 = 2.4	8 – 5.2 = 2.8	11.6 – 8 = 3.6	11.6
External Loans	2	3.6 – 2 = 1.6	6.4 – 3.6 = 2.8	8.4 – 6.4 = 2	8.4
Total	11.2	10	12.8	12.4	46.4

1. c

From 1984-85 to 1985-86, the capital receipts increased from 10 to 12.8 i.e. a percentage increase of 28%

2. b

We need to find 13.6/46.4 as a percentage.

10% of 46.4 is 4.64. Taking multiples of 4.64 and reaching as close to 13.6, we see than 4.64 × 3 = 13.92 and hence our answer will be just less than 30%. How much less? As much as 0.32/46.4.

Next, 1% of 46.4 = 0.464. Thus the answer will be less than 30% by something like 0.7%. Thus answer will be in the vicinity of 29.3%

3. c

In 1983-84, the percent is 2/11.2 i.e. 5/28 which is between 5/25 and 5/30 i.e. between 20% and 16.66%

In 1984-85, the percent is 1.6/10 i.e. 16%

In 1985-86, the percent is 2.8/12.8 i.e. 7/32, which is more than 20%

In 1986-87, the percent is 2/12.4 i.e. 5/31, which is less than 20%.

Thus, highest percentage is for 1985-86

4. d

Whichever bar-segment of 1986-87 is the smallest, will account for the least percentage. Thus, checking the sizes of top-most segment of each bar, we see that external loans is the least and it will account for the least percentage.

5. c

Share of market borrowings in capital receipts in 1983-84 and 84-85 is $\frac{4}{11.2}$ and $\frac{2.4}{10}$. The ratio of this (1984-85 to that of 1983-84) is $\frac{24}{100}\times\frac{112}{40}$ i.e. $\frac{3}{25}\times\frac{28}{5}$ i.e. $\frac{74}{125}$

Thus, the share in 1984-85 decreased by 1 – 84/125 i.e. 41/125. Multiplying by 8, we get the ratio to be 328/1000 i.e. 32.8%

Exercise 31:

1. b

Even though we do not know the total revenue receipts for any year, yet we can order the avenues in order of their percentage increases from one year to other. The order will be same as the order arrived by taking the ratio of their shares in the two years.

We even need not work with numbers. Just checking visually the segments of the 1st bar and the 4th bar, we realise that bar-segment corresponding to Customs has doubled and others have not. So percentage increase in Customs is the highest.

2. b

I. The revenues from customs may be more or may not be more. It would depend on the total receipts of 1980-81 and 1970-71. Thus, this statement is neither necessarily true nor false.

II. As found in Q. 1, the percentage increase in Customs is the highest. Hence this statement is necessarily true.

3. b

Share of Excise in total receipts in 1980-81 is (68 – 46) i.e. 22% and share of Excise in 1990-91 is (62 – 46) i.e. 16%.

If the total receipts of 1980-81 and that of 1990-91 is assumed as x and y respectively, then we have $\frac{16\%\times y}{22\%\times x}=\frac{9}{10}\Rightarrow\frac{y}{x}=\frac{99}{80}$.

Share of Interest in total receipts in 1980-81 and in 1990-91 has remained constant at 12%. This means the percentage change in Interest receipts is same as the percentage change in total receipts. Thus, required percentage is 1 – 99/80 i.e. 19/80 i.e. 23.75%

4. b

If the total receipts of 1990-91 and that of 2000-01 is assumed as x and y respectively, then we have $\frac{12\%\times y}{16\%\times x}=\frac{9}{8}\Rightarrow\frac{y}{x}=\frac{3}{2}$

And we need to find the ratio, $\frac{12\%\times x}{24\%\times y}$. This will be

$\frac{1}{2}\times\frac{2}{3}=\frac{1}{3}$

5. d

Focussing only on 1980-81 to 1990-91, since excise duty has increased we have 16% of y > 22% of x, where x and y are the total revenues in 1980-81 and 1990-91. Thus, $y>\frac{11}{8}x$ i.e. for revenue from excise to increase from 1980-81 to 1990-91, the total revenue has to increase by atleast 37.5%.

The share of Cutoms in the two years increased from 20% to 26% i.e. a percentage increase of 30%

Thus, the minimum percentage increase in revenue from customs over the two years is 37.5 + 30 + (37.5 × 30)/100 i.e. 67.5 + 11.25 i.e. 78.75%

Exercise 32:

In this exercise each corresponding bar refers to the same export, though one is in $ and one is in Rs. Thus, from each pair of corresponding bars, we get the equivalence of dollars and rupees. This is explained for one year and then the calculated values are given for rest of the four years.

For year 2008-09: Rs. 770,000 × 10^7 = $ 172 × 10^9.

Thus, Rs. 1 = $\frac{172}{7700}$ $ or else 1 $ = Rs. $\frac{7700}{172}$

The first ratio can be found by first dividing by 7 and then by 11 as $\frac{172}{7700} = \frac{24.56}{1100} = \frac{2.23}{100} =$ 0.0223 i.e. Rs. 1 = 0.0223$

The second ratio is the reciprocal of this i.e. $\frac{100}{2.23}$

Since 1/22 is 0.4545 and 1/23 is 0.435, the value would like somewhere between 43.5 and 45.4. But for a more precise value, one would need to do quite a bit of calculation, as follows:

Alternately finding the second ratio directly, i.e. finding 77/172:

0.1 × 172 = 17.2. The multiple of 17.2 closest to 77 is the 4th multiple, 17.2 × 4 = 68.8

We yet have to account for 77 – 68.8 i.e. 8.2

0.01 × 172 = 1.72. The multiple of 1.72 closest to 8.2 is the 4th multiple. 1.72 × 4 = 6.88.

We yet have to account for 8.2 – 6.88 i.e. 1.32. This can be thought of orally as 0.172 × 7.

Thus, 77/172 will be 0.447 and hence 1$ = Rs. 44.7

The value of 1 Re and of 1$, for each of the years is given in the following table:

	Value of 1 Re in terms of $	Value of 1 $ in terms of Rs.
2008-09	0.0223	44.77
2007-08	0.0246	40.63
2006-07	0.0225	44.53
2005-06	0.0213	47.00
2004-05	0.0227	44.05

1. c

As read from the table above, the answer will be 100 × 0.0213 = 2.13$

2. b

Read from the table directly as 2007-08

3. a

The greatest increase is from 2007-08 to 1008-09 and also the least base is over this period. Thus, the value of dollar increased by the largest percentage in 2008-09.

4. d

Short-cut:

Exports in $ = Exports in Rs × $/Re

Exports in Rs. has increased by 10%.

Value of $ i.e. Re/$ (rupees that each dollar fetches) decreases by 5% i.e. becomes 19/20 times. Thus, $/Re will become 20/19 times.

Thus, the percentage increase in exports in dollars can be found from $\frac{11}{10} \times \frac{20}{19} = \frac{22}{19}$. The percentage increase will be 3/19 i.e. 3 × 5.26 = 15.78%

If the above short-cut is not understood, assume exports in $year_1$ is Rs. 100. In next year it will become Rs. 110.

Also assume that 1 $ = Rs. 100 in first year. This means in dollar terms exports in $year_1$ is 1 $.

Next year, since value of dollar decreases by 5%, 1 $ = Rs. 95.

Now, we have to convert Rs. 110 to dollars and the exchange rate is 1$ = Rs. 95 or 1Rs = 1/95 $

Thus, Rs. 110 will be $\frac{110}{95} = \frac{22}{19} =$ 1.1578 $

Thus, in dollar terms exports have increased from 1$ to 1.1578$ i.e. a percentage increase of 15.78%

5. c

If exports, in both dollar terms and as well as rupee terms increase by the same percentage, then the value of either currency in terms of the other, remains the same as the previous year. Thus, in 2003-04, the value of dollar would be the same as the value in 2004-05 i.e. Rs. 44.05

Answer Key:

Exercise 1:

1. b 2. d 3. b 4. d 5. a 6. c 7. a 8. a

Exercise 2:

1. d 2. a 3. b 4. c 5. c 6. b 7. c 8. b

Execise 3:

1. a 2. c 3. d 4. b 5. c 6. a

Exercise 4:

1. b 2. b 3. d 4. d 5. b 6. c 7. b 8. a
9. c 10. a

Exercise 5:

1. b 2. d 3. c 4. b 5. c 6. d 7. b 8. d

Calculation Exercise:

1. d 2. b 3. c 4. b 5. b 6. b 7. a 8. c

9. c 10. d 11. a 12. d 13. d 14. b 15. c 16. a

17. c 18. d 19. a 20. b 21. b 22. d 23. c 24. b

25. c 26. a 27. b 28. d 29. b 30. a

Exercise 6:

1. b 2. a 3. c 4. a 5. c 6. c

Exercise 7:

1. a 2. d 3. c 4. b 5. b 6. d

Exercise 8:

See Explanations

Exercise 9:

Answers given and explained in theory after exercise

Exercise 10:

1. a 2. c 3. b 4. a 5. d 6. b 7. b 8. b
9. a 10. d

Exercise 11:

See Explanations

Exercise 12:

1. b 2. c 3. b 4. c 5. d 6. b 7. a 8. c

Exercise 13:

1. c 2. b 3. a 4. c 5. c 6. d

Exercise 14:

1. d 2. d 3. b 4. d 5. a

Exercise 15:

1. c 2. b 3. a 4. a 5. a

Exercise 16:

1. c 2. d 3. a 4. c 5. b 6. b

Exercise 17:

1. c 2. a 3. b 4. c 5. c 6. a 7. c 8. b

Exercise 18:

1. d 2. c 3. b 4. d 5. b 6. c 7. d 8. c

Exercise 19:

1. b 2. c 3. a 4. a 5. c

Exercise 20:

1. b 2. c 3. a 4. d 5. b 6. b

Exercise 21:

1. b 2. d 3. b 4. b 5. c

Exercise 22:

1. d 2. d 3. b 4. c 5. a 6. a 7. d 8. a

Exercise 23:

1. b 2. a 3. a 4. c 5. c

Exercise 24:

1. c 2. c 3. b 4. d 5. a

Exercise 25:

1. d 2. c 3. a 4. b 5. d

Exercise 26:

1. a 2. b 3. d 4. c 5. c

Exercise 27:

1. b 2. c 3. b 4. b 5. b

Exercise 28:

1. a 2. d 3. a 4. c 5. d

Exercise 29:

1. d 2. a 3. c 4. b 5. c

Exercise 30:

1. c 2. b 3. c 4. d 5. c

Exercise 31:

1. b 2. b 3. b 4. b 5. d

Exercise 32:

1. c 2. b 3. a 4. d 5. c